Chapter 10

Mean Structures and Latent Growth Models

I can't understand why people are frightened of new ideas.
I'm frightened of the old ones.

—John Cage

Overview

- Introduction to mean structures
- Identification of mean structures
- Estimation of mean structures
- Structured means in measurement models
- Latent growth models

Introduction to mean structures

- If only covariances are analyzed, then it is assumed that the means of all variables equal zero
- Sometimes this assumption is too restrictive, such as when repeated measures variables are analyzed and means are expected to change
- Means are estimated in SEM by adding a *mean structure* to the model's basic covariance structure (i.e., its structural or measurement components)

Introduction to mean structures

- The input data for the analysis of a model with a mean structure are covariances and means (or the raw scores)

- The SEM approach to the analysis of means is distinguished by the capability to test hypotheses about the

    1. means of latent variables

    2. covariance structure of the error terms

- In contrast, other statistical methods, such as the analysis of variance (ANOVA), are concerned mainly with the means of observed variables and are not as flexible in the analysis of error covariances

Introduction to mean structures

- The representation of means in structural equation models is based on the general principles of regression
- These principles are related to how computer programs for multiple regression calculate intercepts (i.e., the constant) of regression equations
- Recall that intercepts reflect the unstandardized regression coefficients *and* the means of all variables

Introduction to mean structures

- For example, given the following equation for the regression of *Y* on *X*:

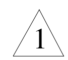
$$\hat{Y} = B\,X + A$$

the intercept can be expressed as:

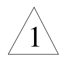
$$A = M_Y - B\,M_X$$

- The term *B* can be seen as the covariance structure of the equation and the term *A* as the mean structure

Introduction to mean structures

- A multiple regression computer program calculates intercepts and means by regressing variables on a constant
- This constant equals 1.0 for each case and it is represented in model diagrams here with the symbol 

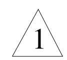
Introduction to mean structures

- The symbol 

is from the McArdle-McDonald symbolism for SEM
- However, it is not a standard symbol in SEM
- In fact, some authors do not use a special symbol in model diagrams when means are analyzed
- It is used here to explicitly designate the analysis of means

Introduction to mean structures

- General principles about the estimation of intercepts and means in multiple regression:

  1. When a criterion (e.g., *Y*) is regressed on a predictor and a constant (e.g., *X*, ), the unstandardized coefficient for the constant is the intercept

  2. When a predictor (e.g., *X*) is regressed on a constant, the unstandardized coefficient is the mean of the predictor

Introduction to mean structures

- Path-analytic principles about the estimation of intercepts and means:

  1. The mean of an endogenous variable (e.g., *Y*) is a function of three parameters: the

     a. intercept

     b. unstandardized path coefficient

     c. mean of the exogenous variable (e.g., *X*)

Introduction to mean structures

- Path-analytic principles about the estimation of intercepts and means:

  2. The model-implied (predicted) mean for an observed variable is the total effect of the constant on that variable

     a. For exogenous variables, the unstandardized path coefficient for the direct effect of the constant (which is also a total effect) is a mean

     b. For endogenous variables, the direct effect of the constant is an intercept, but the total effect is a mean

Identification of mean structures

- The parameters of a structural equation model with a mean structure include the

    1. means of the exogenous variables
    2. intercepts of the endogenous variables
    3. the number of parameters in the covariance portion of the model counted in the usual way for that type of model

Identification of mean structures

- A simple rule for counting the total number of observations available to estimate the parameters of model with a mean structure is

$$v\,(v + 3)/2$$

where v is the number of observed variables

Identification of mean structures

- In order for a mean structure to be identified, the number of its parameters (i.e., exogenous variable means, endogenous variable intercepts) cannot exceed the total number of means of the observed variables

- The identification status of a mean structure must be considered separately from that of the covariance structure

- That is, an overidentified covariance structure will not identify an underidentified mean structure and vice-versa

Identification of mean structures

- If the mean structure is just-identified, it has just as many free parameters as observed means—therefore the

    1. model-implied means (i.e., total effects of the constant) will exactly equal the corresponding observed means

    2. fit of the model with just the covariance structure will be identical to that of the model with both the covariance structure and the mean structure

- It is only when the mean structure is overidentified that the predicted means could differ from the observed means

- That is, one or more *mean residuals* may not equal zero

Estimation of mean structures

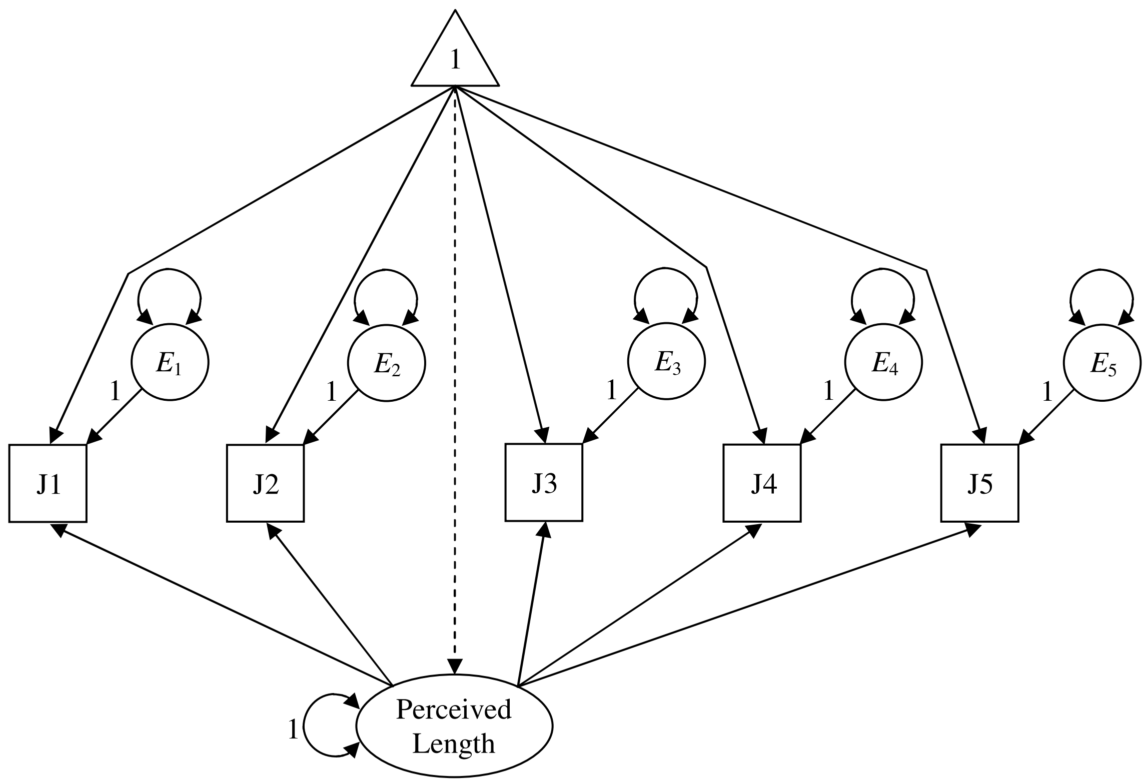
- The most general estimation method in SEM, maximum likelihood (ML), can also be used to analyze means
- However, not all standardized fit indexes for models with covariance structures only may be available for models with both covariance and mean structures
- This is especially true for incremental fit indexes, such as the CFI, that measure the relative improvement in the fit of the researcher’s model over a null model

Estimation of mean structures

- When only covariances are analyzed, the null model for incremental fit indexes is typically the independence model, which assumes zero population covariances
- However, the independence model for an incremental fit index is more difficult to define when both covariances and means are analyzed
- For example, an independence model where all covariances and means are fixed to equal zero may be very unrealistic
- An alternative independence model allows for the means of the observed variables to be freely estimated (i.e., they are not assumed to be zero)

Structured means in measurement models

- A standard CFA model assumes that the means of all variables are zero
- Example: A CFA model with a mean structure that relaxes this assumption for some variables (Figure 10.2):



Structured means in measurement models

- Based the principles discussed earlier, the direct effects of the constant in the model just presented on the

    1. endogenous indicators should be intercepts for the regression of the indicators on the factor (but total effects will be means)

    2. exogenous factor should be the factor mean

Structured means in measurement models

- However, the CFA model with structured means just presented is not identified if estimated in a single sample

- This is because there are five observed means but six parameters of the mean structure (i.e., it is under-identified)

- One strategy is to assume that the mean of the factor is zero (i.e., △1 → Perceived Length = 0) and estimate only the intercepts of the indicators

Structured means in measurement models

- In general, factor means can be estimated if a CFA model

    1. is analyzed across multiple samples, and

    2. constraints are imposed on certain parameter estimates

Structured means in measurement models

- The most common strategy is to fix the factor means to zero in one of the groups, which establishes that group as a reference sample

- The factor means in the other samples are freely estimated, and their values estimate the relative differences on the factor means compared with the reference sample

- Additional constraints may also include cross-group equality constraints on the factor loadings and intercepts, which together test for invariance in measurement and in regressions of the indicators on the factors (chap. 11)

Latent growth models

- The term *latent growth model* (LGM) refers to a class of models for longitudinal data that can be analyzed in SEM or other statistical techniques, such as hierarchical linear modeling (HLM)
- Because the identification requirements for a LGM are different than for a CFA model with structured means, the former can be analyzed in a single sample

Latent growth models

- The particular kind of LGM outlined here

    1. has been described by several different authors (e.g., T. Duncan, S. Duncan, Strycker, Li, & Alpert, 1999)

    2. is specified as a structural regression model with a mean structure

    3. can be analyzed with standard SEM software

Latent growth models

- The analysis of a LGM in SEM generally requires

    1. a continuous dependent variable measured on at least three different occasions

    2. scores that have the same units across time, can be said to measure the same construct at each assessment, and are *not* standardized

    3. data that are *time structured*, which means that cases are all tested at the same intervals

- These intervals need not be equal—for example, a sample of children may be observed at 3, 6, 12, and 24 months of age

- But all cases must be tested at the same intervals

Latent growth models

- The raw scores are not generally required to analyze a LGM
- That is, such models can often be analyzed with matrix summaries of the data
- These matrix summaries must include the covariances (or correlations and standard deviations) and means of all variables
- However, Willett and Sayer (1994) noted that that inspection of the *empirical growth record*—the raw scores for each case—can help to determine whether it may be necessary to model nonlinear growth

Latent growth models

- Latent growth models can be described as a special kind of *multilevel model* for *hierarchical data*
- In this case, the latter refers to panel data where individuals are observed over time and repeated measures are nested within each individual (Hser, Chou, Messer, & Anglin, 2001)

Latent growth models

- Scores from the same case are probably not independent, and this lack of independence should be taken into account in the statistical analysis
- Other kinds of hierarchical data structures arise when individuals are clustered into larger units such as students within classrooms or siblings within families
- Multilevel structural equation models other than latent growth models are considered later (chap. 13)

Latent growth models

- Latent growth models are often analyzed in two steps
- The first step involves the analysis of a *change model* that involves just the repeated measures variables
- A change model attempts to explain the covariances and means of these variables
- Given an acceptable change model, the second step adds variables to the model that may *predict change* over time
- This two-step approach makes it easier to identify potential sources of poor model fit compared with the analysis of a prediction model in a single step

Latent growth models

- A basic change model has these characteristics:

  1. Each repeated measures variable is specified as an indicator of two latent growth factors, Initial Status (IS) and Linear Change (LC)

     a. The IS factor represents the baseline level, and all unstandardized loadings on this factor equal 1.0

     b. The LC factor represents rate of linear change, and loadings on it may be fixed to constants that correspond to the times of measurement

     c. One of these fixed loadings on LC will be zero, which establishes that particular measurement as the baseline level

Latent growth models

- A basic change model has these characteristics:

  2. The IS and LC factors are specified to covary, and the estimate of this covariance indicates the degree to which initial levels predict rates of subsequent linear change

Latent growth models

- A basic change model has these characteristics:

  3. The model will have a mean structure in which the constant has direct effects on both latent growth factors

     a. The mean of the IS factor is the average initial level on whatever characteristic is measured over time, adjusted for measurement error

     b. The mean of the LC factor reflects the average amount of linear change over time, also adjusted for measurement error

     c. The variances of the IS and KC factors reflect the degree of individual differences in, respectively, the average initial level and rate of linear change

Latent growth models

- A basic change model has these characteristics:

  4. It is possible to model the error covariance structure by specifying a pattern of correlated errors

Latent growth models

- The ability to model error covariances is one thing that really differentiates SEM from traditional statistical techniques for repeated measures data—for example:

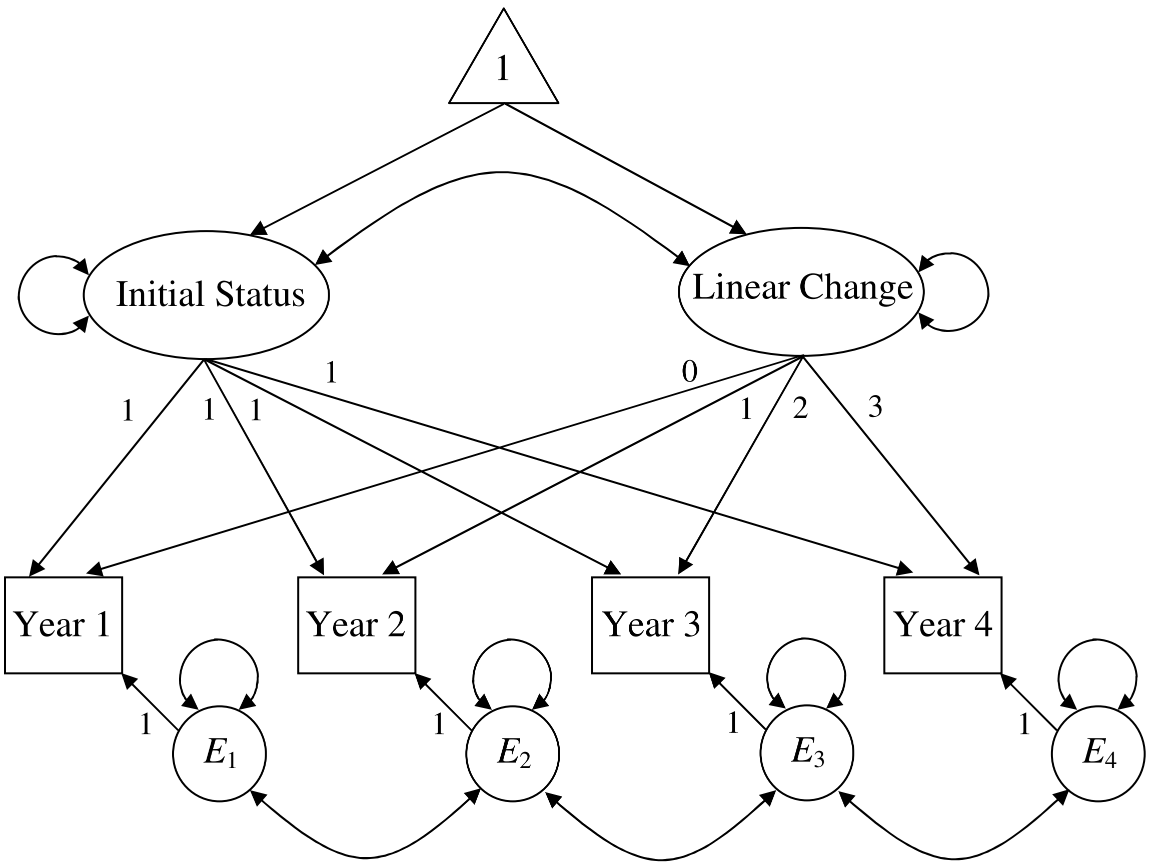
  1. ANOVA assumes that the error variances of repeated measures variables are equal and independent

  2. MANOVA does not assume independent errors, but neither ANOVA or MANOVA directly analyze means of latent variables

  3. Both of these traditional techniques also treat individual differences in growth trajectories as error variance (see Cole, Maxwell, Avery, & Salas, 1993)

Latent growth models

- Example of a change model for annual survey data collected by S. Duncan and T. Duncan (1996; Figure 10.2):



Latent growth models

- With an adequate model of change in hand, a model that predicts this change can then be analyzed
- Potential predictors of change are added to a basic change model by

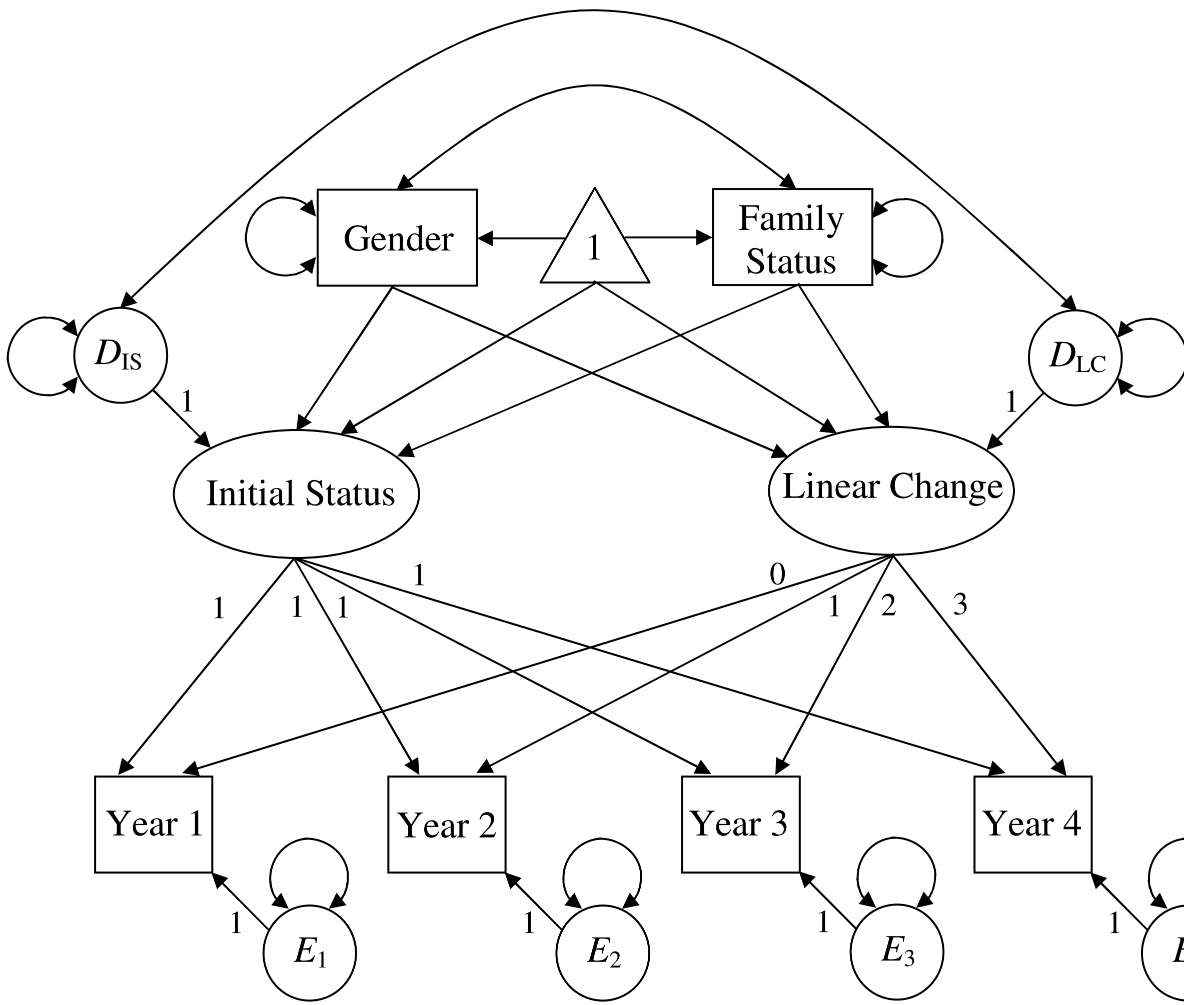
    1. including them in the mean structure
    2. regressing the latent growth factors (e.g., IS, LC) on the predictors

Latent growth models

- The latent growth factors are endogenous in a change model, which means that each will have a disturbance
- These disturbances are typically specified as correlated
- The prediction model is also a MIMIC (multiple indicators and multiple causes) model because the factors have both effect and cause indicators

Latent growth models

- Example of a prediction model for annual survey data collected by S. Duncan and T. Duncan (1996; Figure 10.3):



Latent growth models

- The basic framework just discussed for univariate growth curve modeling in a single sample can be extended in many ways—some examples:

  1. The predictors can be *time-variant* (e.g., gender, family status) or *time-varying*, which means that they are also repeated measures variables (e.g., Kaplan, 2000, pp. 155-159)

  2. Measurement error of predictors that are single-indicators can be taken into account using the same basic method as for structural regression models

  3. Predictors can be latent variables each measured by multiple indicators, that is, the prediction part of the model can be fully latent (e.g., Chan, 1998)

Latent growth models

- Other extensions of the basic framework for univariate growth curve modeling:

    1. It may also be possible within the limits of identification to specify that some loadings on a latent growth factor that does not represent initial status are free parameters

        a. Example: Estimate an empirical growth function where one loading is fixed to zero—which sets the initial level—and another is fixed to 1.0—which scales the factor—but all others are freely estimated

        b. Ratios of freely-estimated loadings for this empirical growth function can also be formed to compare rates of development at different points in time

Latent growth models

- Other extensions of the basic framework for univariate growth curve modeling:

    2. Analyze multivariate latent growth models of change across two or more domains in the same sample (e.g., Curran, Harford, & B. Muthén, 1996)

        a. This refers to the analysis of a model of *cross-domain change*

        b. Estimates whether initial status in one domain predicts rate of change in another domain and vice-versa

    3. Analyze a LGM across multiple samples (e.g., Willett & Sayer, 1996)

References

Chan, D. (1998). The conceptualization and analysis of change over time: An integrative approach incorporating longitudinal mean and covariance structures analysis (LMACS) and multiple indicator latent growth modeling (MLGM). *Organizational Research Methods*, *1*, 421-483.

Cole, D. A., Maxwell, S. E., Avery, R., & Salas, E. (1993). Multivariate group comparisons of variable systems: MANOVA and structural equation modeling. *Psychological Bulletin*, *114*, 174-184.

Curran, P. J., Harford, T. C., & Muthén, B. O. (1996). The relation between heavy alcohol use and bar patronage: A latent growth model. *Journal of Studies on Alcohol*, *57*(4), 410-418.

Duncan. S. C., & Duncan, T. E. (1996). A multivariate latent growth curve analysis of adolescent substance use. *Structural Equation Modeling*, *3,* 323-347.

Duncan, T., Duncan, S., Alpert, A., Hops, H., Stoolmiller, M., & Muthén, B. (1997). Latent variable modeling of longitudinal and multilevel substance abuse data. *Multivariate Behavioral Research*, *32*, 275-318.

Hser, Y.-I., Chou, C.-P., Messer, S. C., & Anglin, M. D. (2001). Analytic approaches for assessing long-term treatment effects: Examples of empirical applications and findings. *Evaluation Review*, *25*, 233-262.

Kaplan, D. (2000). *Structural equation modeling*. Thousand Oaks, CA: Sage.

Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin*, *116*, 363-381.

Willett, J. B., & Sayer, A. G. (1996). Cross-domain analyses of change over time: Combining growth modeling and covariance structure analysis. In G. A. Marcoulides and R. E. Schumacker (Eds.), *Advanced structural equation modeling* (pp. 125-157). Mahwah, NJ: Erlbaum.