# Chapter 9

## *Nonrecursive Structural Models*

The best way to predict the future is to invent it.

—Alan Kay

## Overview

- Specification of nonrecursive models
- Identification of nonrecursive models
- Estimation of nonrecursive models

# Specification of nonrecursive models

- Recursive structural models assume that

    1. all causal effects are represented as unidirectional

    2. there are no disturbance correlations between endogenous variables with direct effects between them

- These assumptions are very restrictive

- For example, many “real world” causal processes are based on cycles of mutual influence, that is, feedback

- The presence of a feedback loop in a structural model automatically makes it nonrecursive

# Specification of nonrecursive models

- A feedback loop involves mutual causation among variables measured at the same time (i.e., the design is cross-sectional)
- Recall that

  1. direct feedback involves only two variables in a reciprocal relation (e.g., $Y_1 \rightleftarrows Y_2$)

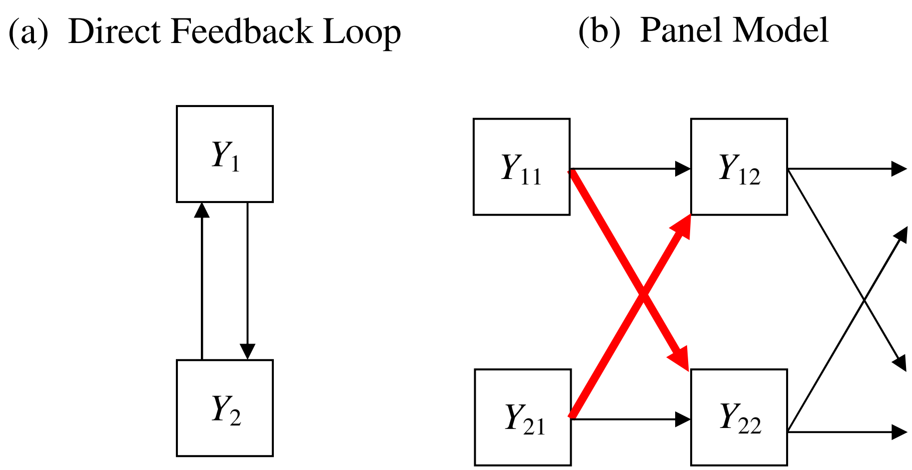
  2. indirect feedback involves three or more variables (e.g., $Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_1$)

# Specification of nonrecursive models

- An alternative way to estimate reciprocal causal effects requires a longitudinal design where $Y_1$ and $Y_2$ are each measured at two or more different points in time

- This type of design is a *panel design*, and reciprocal causation is estimated by *cross-lagged direct effects* between $Y_1$ and $Y_2$ measured at different times (red below)

- Example: Figure 9.1:



# Specification of nonrecursive models

- A complete panel model may be recursive or nonrecursive depending on its pattern of disturbance correlations
- The analysis of a panel model is not a panacea for estimating reciprocal causal effects (Kenny, 1979)
- For example, longitudinal designs are more expensive and subject to loss of cases over time
- It can also be difficult to specify measurement occasions that match actual finite causal lags

# Specification of nonrecursive models

- Panel designs are also *not* generally useful for resolving causal priority between reciprocally-related variables unless some highly restrictive (i.e., unrealistic) assumptions are met

- One of these is *stationarity*, the requirement that the causal structure does not change over time

- Maruyama (1998) notes that the general requirement that there are no omitted causes correlated with those in the model is even more critical for panel models because of the repeated sampling of variables over time

# Specification of nonrecursive models

- The complexity of panel models can also increase rapidly as more variables in the form of repeated measurements are added to the model (e.g., Cole & Maxwell, 2003; Finkel, 1995)

- For many researchers, the estimation of reciprocal causation between variables measured at the same time is the only realistic alternative to a longitudinal design

## Specification of nonrecursive models

- Kaplan, Harik, and Hotchkiss (2001) remind us that data from a cross-sectional design give only a “snapshot” of an ongoing dynamic process
- Therefore, the estimation of reciprocal effects with cross-sectional data requires the assumption of *equilibrium*
- This means that any changes in the system that underlies a presumed feedback relation have already manifested their effects and that the system is in a steady state
- That is, estimates of the direct effects that make up a feedback loop (e.g., $Y_1 \rightarrow Y_2$ and $Y_2 \rightarrow Y_1$ of $Y_1 \rightleftarrows Y_2$) do not depend on the particular time point of data collection

# Specification of nonrecursive models

- Recall that the presence of a disturbance correlation reflects the assumption that the corresponding endogenous variables share at least one common omitted cause
- The disturbances of variables involved in feedback loops are often specified as correlated
- This specification also often makes sense because if variables are presumed to mutually cause each one another, then it seems plausible to expect that they may have common omitted causes
- The presence of disturbance correlations in particular patterns in nonrecursive models also helps to determine their identification status

# Identification of nonrecursive models

- Assume that the same two necessary identification requirements for any kind of structural equation model are met: Every latent variable has a scale and $df_M \geq 0$

- The next few diagrams illustrate path models, but the same principles apply to the structural components of structural regression models

# Identification of nonrecursive models

- Unlike recursive models, nonrecursive models are not always identified
- There are algebraic means to determine whether the parameters of a nonrecursive model can be expressed as unique functions of its observations (e.g., Berry, 1984, pp. 27-35)
- However, these techniques are practical only for very simple models

# Identification of nonrecursive models

- There are alternatives that involve checking whether a nonrecursive model meets certain requirements for identification that can be readily evaluated by hand
- Some of these requirements are only necessary for identification, which means that satisfying them does not guarantee that the model is actually identified
- If a nonrecursive structural model satisfies a sufficient condition, however, then it is identified

# Identification of nonrecursive models

- The nature and number of conditions for identification that a nonrecursive model must satisfy depend on its pattern of disturbance correlations

- Specifically, the necessary *order condition* and the sufficient *rank condition* apply to models with unanalyzed associations between all pairs of disturbances either

    1. for the whole model or

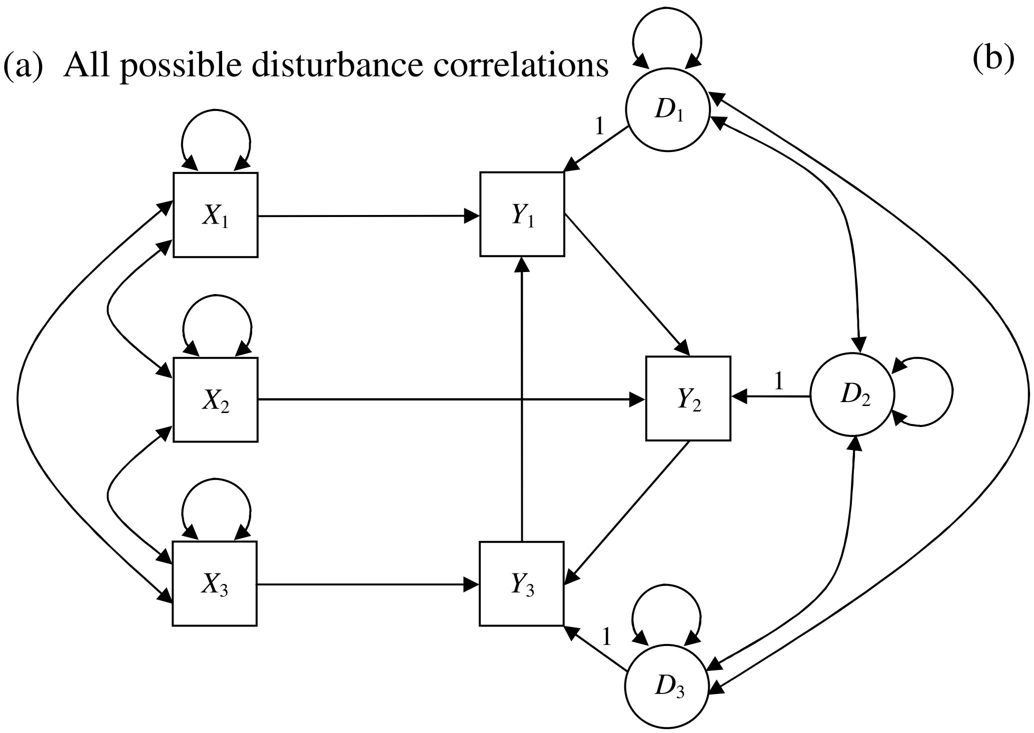
    2. within blocks of endogenous variables that are recursively related to each other

- Some authors use the term *block recursive* to describe nonrecursive models with the second pattern of disturbance correlations

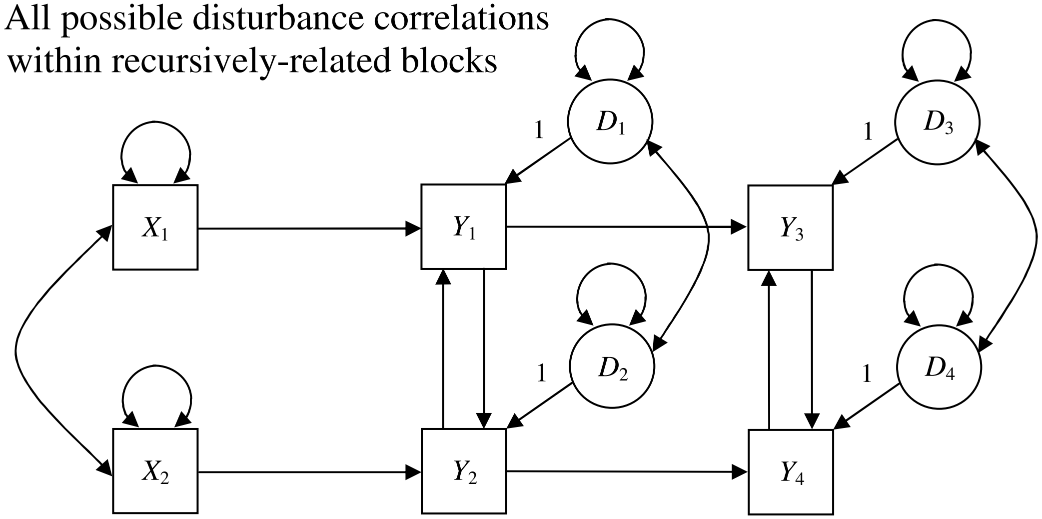
# Identification of nonrecursive models

- Examples of nonrecursive models with disturbance correlations between all pairs of endogenous variables (left; Figure 9.2(a)) and within sets of recursively-related blocks (right; Figure 9.2(b)):

(a) All possible disturbance correlations




(b) All possible disturbance correlations within recursively-related blocks



# Identification of nonrecursive models

- If a nonrecursive structural model has either no disturbance correlations or less than all possible disturbance correlations such that the model is not block recursive, the order and rank conditions are too conservative

- That is, such “none-of-the-above” nonrecursive models that fail either condition could nevertheless be identified

# Identification of nonrecursive models

- Unfortunately, there may be no sufficient condition that can be readily evaluated by hand to determine whether a "none-of-the-above" nonrecursive model is actually identified

- However, such models can be evaluated with the methods outlined in the previous chapter for confirmatory factor analysis (CFA) models for which there is no easily-applied sufficient condition for identification

- These methods involve checking whether a converged solution from a SEM computer program passes empirical tests of uniqueness, but recall that such tests are only necessary conditions for identification

# Identification of nonrecursive models

- The *order condition* is a counting rule applied to each endogenous variable in a nonrecursive model with either all possible disturbance correlations or that is block recursive

- If the order condition is not satisfied, the equation for that endogenous variable is underidentified

- One evaluates the order condition by tallying the numbers of variables in the structural model (except disturbances) that have direct effects on each endogenous variable versus the number that do not

- Let us call the latter *excluded variables*

# Identification of nonrecursive models

- The order condition requires that:

  *The number of excluded variables for each endogenous variable equals or exceeds the total number of endogenous variables minus 1*

- For models with all possible disturbance correlations, the total number of endogenous variables equals that for the whole model

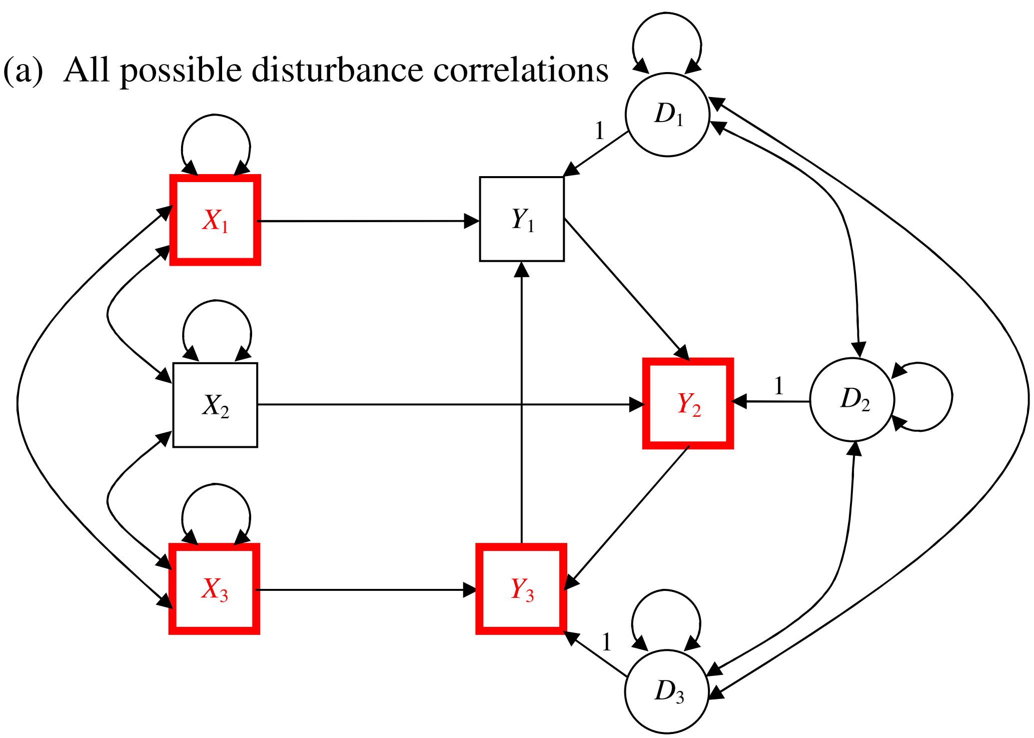
- For nonrecursive models that are block recursive, however, the total number of endogenous variables is counted separately for each block when the order condition is evaluated.

# Identification of nonrecursive models

- Example: A total of three variables are excluded from the equation of each endogenous variable in Figure 9.2(a)—which exceeds the minimum number (3 – 1 = 2)—so the order condition is satisfied

- Endogenous variable $Y_2$ and all its excluded variables are shown in red:

(a) All possible disturbance correlations



# Identification of nonrecursive models

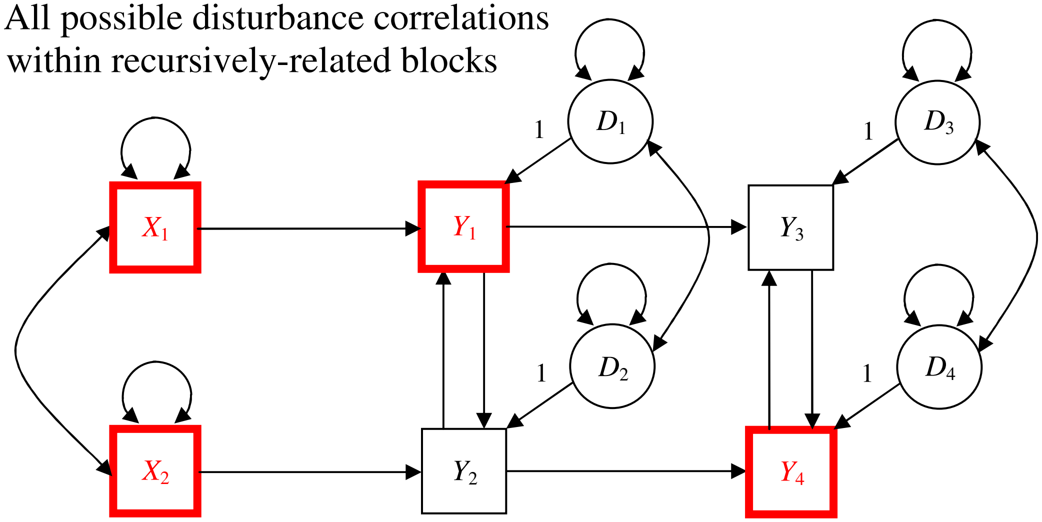
- Example: There are two recursively-related blocks of endogenous variables in the model of Figure 9.2(b)

- Each block has two variables, so the total number of endogenous variables for each block is 2

- To satisfy the order condition, there must be at least 2 – 1 = 1 variables excluded from the equation of each endogenous variable in both blocks, which is here true

- Endogenous variable $Y_4$ and all its excluded variables are shown in red:

(b) All possible disturbance correlations within recursively-related blocks



# Identification of nonrecursive models

- Because the order condition is only necessary, we still do not know whether the nonrecursive models of Figure 9.2 are actually identified
- However, evaluation of the sufficient *rank condition* will provide the answer
- The rank condition is usually described in the SEM literature in matrix algebra terms (e.g., Bollen, 1989, pp. 98-103)
- Berry (1984) devised an algorithm for checking the rank condition that does not require extensive knowledge of matrix operations, a simpler version of which is presented here

# Identification of nonrecursive models

- For nonrecursive models with all possible disturbance correlations, the rank condition can be viewed as a requirement that each variable in a feedback loop has a unique pattern of direct effects on it from variables outside the loop
- Such a pattern of direct effects provides a sort of “statistical anchor” so that the parameters of variables involved in feedback loops can be estimated distinctly from one another
- However, this analogy does not hold for models considered in this book to be nonrecursive that do not have feedback loops
- Therefore, a more formal method of evaluating the rank condition is needed

# Identification of nonrecursive models

- The starting point for checking the rank condition is to construct something that Berry (1984) called a *system matrix*

- In a system matrix, the endogenous variables of the structural model are listed on the left side of the matrix (rows) and all variables (excluding disturbances) along the top (columns)

# Identification of nonrecursive models

- In each row of a system matrix, a “0” or “1” appears in the columns that correspond to that row:

  1. A “1” indicates that the variable represented by that column has a direct effect on the endogenous variable represented by that row

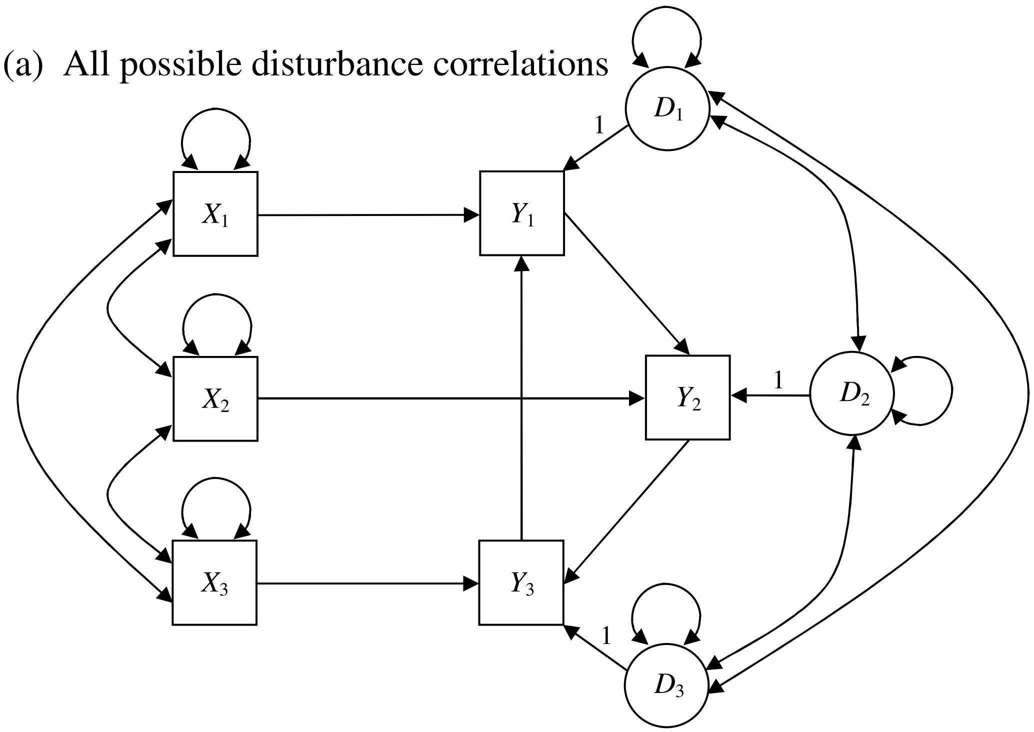
  2. A “1” also appears in the column that corresponds to the endogenous variable represented by that row

  3. The remaining entries are “0’s,” and they indicate excluded variables

# Identification of nonrecursive models

- The system matrix for the model of Figure 9.2(a) with all possible disturbance correlations (left) is presented here (right):

(a) All possible disturbance correlations




| | $X_1$ | $X_2$ | $X_3$ | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|---|---|---|
| $Y_1$ | 1 | 0 | 0 | 1 | 0 | 1 |
| $Y_2$ | 0 | 1 | 0 | 1 | 1 | 0 |
| $Y_3$ | 0 | 0 | 1 | 0 | 1 | 1 |

# Identification of nonrecursive models

- The rank condition is evaluated using the system matrix
- Like the order condition, the rank condition must be evaluated for the equation of each endogenous variable

# Identification of nonrecursive models

- The steps to evaluate the rank condition for a model with all possible disturbance correlations are as follows:

  1. Begin with the first row of the system matrix. Cross out all entries of that row. Also cross out any column in the system matrix with a “1” in this row. Use the entries that remain to form a new, reduced matrix

  2. Simplify the reduced matrix further by deleting any row with entries that are all zeros. Also delete any row that is an exact duplicate of another or that can be reproduced by adding other rows together. The number of remaining rows is the rank. The rank condition is met for the equation of this endogenous variable if the rank of the reduced matrix is greater than or equal to the total number of endogenous variables minus 1

# Identification of nonrecursive models

- Steps to evaluate the rank condition for a model with all possible disturbance correlations:

    3. Repeat steps 1 and 2 for every endogenous variable. If the rank condition is satisfied for every endogenous variable, then the model is identified

# Identification of nonrecursive models

- Steps 1 and 2 applied to the system matrix for $Y_1$ in the model of Figure 9.2(a) are summarized here:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} X_1 & X_2 & X_3 & Y_1 & Y_2 & Y_3 \end{array} \\
\begin{array}{c} \rightarrow \; Y_1 \\ Y_2 \\ Y_3 \end{array} &
\left[ \begin{array}{cccccc}
\cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} \\
\cancel{0} & 1 & 0 & \cancel{1} & 1 & \cancel{0} \\
\cancel{0} & 0 & 1 & \cancel{0} & 1 & \cancel{1}
\end{array} \right]
\end{array}
\rightarrow
\left[ \begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array} \right]
\rightarrow \text{Rank} = 2
$$

- The rank for the equation of $Y_1$ is 2, which equals the minimum (3 – 1 = 2); thus, this equation satisfies the rank condition
- Because the equations for $Y_2$ and $Y_3$ also satisfy the rank condition (see Table 9.1), this condition is met for the whole model
- Therefore, the entire model is identified

# Estimation of nonrecursive models

- Technical problems are more likely when nonrecursive models are analyzed compared with recursive models—for example:

    1. Iterative estimation of direct effects or disturbance variances-covariances for variables involved in feedback loops may fail without quite accurate start values (see Appendix 5.B)

    2. A converged solution may contain Heywood cases (e.g., negative variance estimates) or other kinds of illogical results (e.g., extremely large estimated standard errors)

    3. Estimation of nonrecursive models may be more susceptible to empirical underidentification, which can happen when estimates of certain key paths are close to zero

# Estimation of nonrecursive models

- The imposition of an equality or proportionality constraint on the direct effects of a feedback loop is one way to reduce the number of free model parameters without dropping paths
- A possible drawback to imposing equality constraints on feedback loops is that doing so precludes the detection of unequal mutual influence
- If equality constraints are blindly imposed when bidirectional effects differ in magnitude, then not only may the model poorly fit the data, but the researcher may also miss an important finding

# Estimation of nonrecursive models

- In contrast, a proportionality constraint allows for unequal mutual influence but on an a priori basis
- Like equality constraints, proportionality constraints reduce the number of free parameters, one for each pair of direct effects
- Also recall that equality or proportionality constraints usually apply only in the unstandardized solution

# Estimation of nonrecursive models

- Variables in feedback loops have indirect effects (and thus total effects) on *themselves*
- For example, an indirect effect of $Y_1$ on itself in the direct feedback loop $Y_1 \rightleftarrows Y_2$ is the sequence $Y_1 \rightarrow Y_2 \rightarrow Y_1$
- Estimation of indirect and total effects among variables in a feedback loop requires the assumption of equilibrium
- However, it is important to realize that there is generally no statistical way to directly evaluate whether the equilibrium assumption is tenable when the data are cross-sectional

# Estimation of nonrecursive models

- Kaplan et al. (2001) note that rarely is the equilibrium assumption explicitly acknowledged in the literature on applications of SEM where feedback effects are estimated with cross-sectional data

- This is unfortunate because the results of a computer simulation by Kaplan et al. (2001) indicate that violation of the equilibrium assumption can lead to severely biased estimates

- Specifically, Kaplan et al. (2001) found that estimates of direct effects in feedback loops can vary dramatically depending on when the simulated data were collected after a dynamic system has been perturbed

# Estimation of nonrecursive models

- Kaplan et al. (2001) also found that the *stability index* did not accurately measure the degree of bias due to lack of equilibrium
- The stability index in printed in the output of some SEM computer programs, such as Amos, when a nonrecursive model is analyzed
- It is based on certain mathematical properties of the matrix of coefficients for direct effects among all the endogenous variables in a structural model, not just those involved in feedback loops
- These properties concern whether estimates of the direct effects would get infinitely larger over time
- If so, the system is said to “explode” because it may never reach equilibrium, given the observed direct effects among the endogenous variables

# Estimation of nonrecursive models

- A standard interpretation of the stability index is that values less than 1.0 are taken as positive evidence for equilibrium, but values greater than 1.0 suggest the lack of equilibrium
- However, this interpretation is *not* generally supported by the Kaplan et al.'s (2001) computer simulations results, which emphasizes the need to evaluate equilibrium on rational grounds

# Estimation of nonrecursive models

- Bentler and Raykov (2000) noted that squared multiple correlations ($R^2_{\text{smc}}$) computed as one minus the ratio of the disturbance variance over the total variance may be inappropriate for endogenous variables involved in feedback loops

- This is because the disturbances of such variables may be correlated with one of its presumed causes, which violates a requirement of least squares estimation that the residuals are unrelated to the predictors

# Estimation of nonrecursive models

- Bentler and Raykov (2000) described a general approach to estimating explained variance in nonrecursive models that corrects for model-implied correlations between disturbances and predictors

- Their proportion of explained variance for nonrecursive models, the *Bentler-Raykov corrected* $R^2$, is automatically calculated by EQS

# Estimation of nonrecursive models

- Maximum likelihood (ML) estimation is commonly used to analyze nonrecursive models
- An older alternative for nonrecursive path models is *two-stage least squares* (2SLS) estimation
- The method of 2SLS is a variation on standard multiple regression (MR) gets around the problem of model-implied correlations between disturbances and presumed causes of endogenous variables

# Estimation of nonrecursive models

- The method of 2SLS is also a

    1. partial-information method that analyzes one structural equation at a time

    2. non-iterative method, which means that it needs no start values

# Estimation of nonrecursive models

- For nonrecursive path models, 2SLS is nothing more than MR applied in two separate stages
- The aim of the first stage is to replace every problematic observed causal variable with a newly-created predictor
- A "problematic" causal variable has a direct effect on an endogenous variable *and* a model-implied correlation with the disturbance of that variable
- Variables known as *instruments* or *instrumental variables* are used to create the new predictors

# Estimation of nonrecursive models

- An instrumental variable has a direct effect on the problematic causal variable, but no direct effect on the endogenous variable (i.e., it is excluded from the equation of the latter)
- Note that both conditions just described are given by theory, not by statistical analysis
- An instrument can be either exogenous or endogenous
- The same variable cannot serve as the instrument for both variables in a direct feedback loop
- Also, one of the variables does not need an instrument if the disturbances of the variables in the loop are specified as uncorrelated (Kenny, 2002)

# Estimation of nonrecursive models

- The 2SLS method is applied as follows:

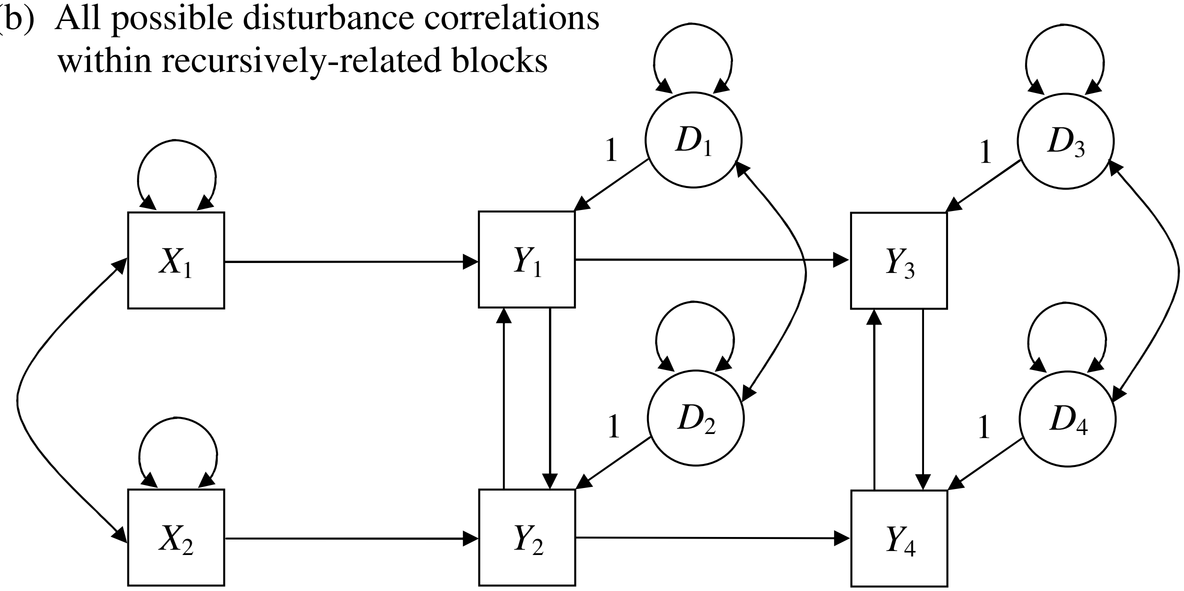
1. The problematic causal variable is regressed on the instrument. The predicted criterion variable from this analysis will be uncorrelated with the disturbance of the endogenous variable and replaces the problematic causal variable

2. When similar replacements are made for all problematic causal variables, the second stage of 2SLS is just a standard MR analysis conducted for each endogenous variable but using the predictors created in the first step whenever the original ones were replaced

# Estimation of nonrecursive models

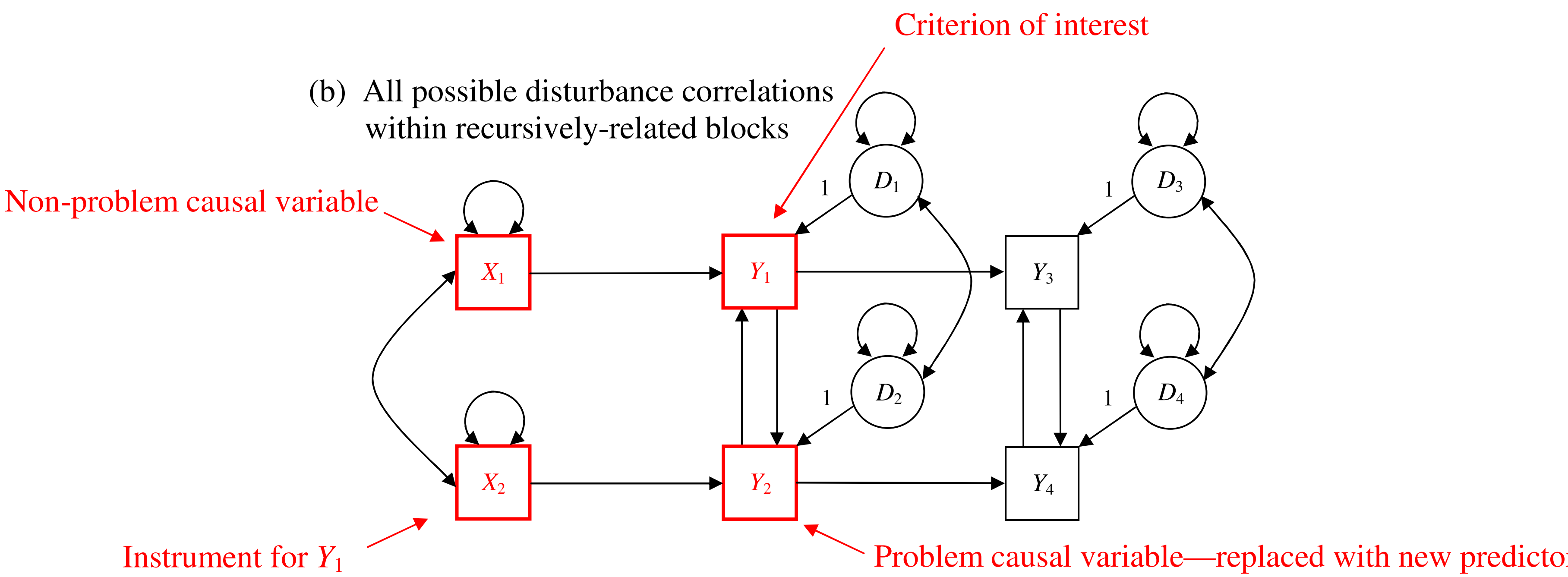
- Example: The nonrecursive path model of Figure 9.2(b) specifies two direct causes of $Y_1$, the variables $X_1$ and $Y_2$:

(b) All possible disturbance correlations within recursively-related blocks



# Estimation of nonrecursive models

- From the perspective of standard MR, $Y_2$ is a problematic causal variable because of the model-implied correlation between this variable and the disturbance of $Y_1$
- The instrument here is $X_2$ because it is excluded from the equation of $Y_1$ and has a direct effect on $Y_2$, the problematic causal variable:



# Estimation of nonrecursive models

- The problem causal variable $Y_2$ is regressed on the instrument $X_2$ in a standard regression analysis
- The predicted criterion variable from this first analysis, $\hat{Y}_2$, replaces $Y_2$ as a predictor of $Y_1$ in a second regression analysis where $X_1$ is the other predictor
- The coefficients from the second regression analysis are taken as the least squares estimates of the path coefficients for the direct effects of $X_1$ and $Y_2$ on $Y_1$

# Estimation of nonrecursive models

- For more information about 2SLS for path models, see
  - Berry (1984, chap. 5)
  - James and Singh (1978)
  - Kenny (1979, pp. 83-92, 103-107; 2002)
- Bollen (1996) describes forms of 2SLS estimation for latent variable models
- LISREL uses a 2SLS-based method to generate start values that are passed to iterative methods, such as ML estimation

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