

Confirmatory Factor Analysis Using *Mplus*

In this document I explain how to use *Mplus* to run confirmatory factor analyses.

The data from this study are responses of 1022 people to items on the Achievement Goal Questionnaire (Elliot & McGregor, 2001) collected as part of a study by Finney, Pieper, and Barron (2004) who were interested in determining whether the factor structure reported by Elliott and McGregor could be replicated in a general, rather than a specific, academic context. The response scale ranged from 1 = not at all true of me to 7 = very true of me. There are no missing data, but I have included the “missing” command used to specify missing data in *Mplus* to demonstrate how this would be done. The data are in tab-delimited format and can be found in the file “goal orientation.dat”

CFA Syntax

Below I present the *Mplus* syntax, followed by an explanation of each command. *Mplus* commands are in boldface. Non-boldface words indicate information that must be provided by the researcher for their specific data.

```
Title: CFA of goal orientation data;  
data: file is goal2.dat;  
      format is free;  
variable: names are i1 i2 i3 i4 i5 i6 i7 i8 i9 i10 i11 i12;  
            missing all (8,9,10);  
model:  
      perfapp by i1 i2 i3;  
      peravoid by i4 i5 i6;  
      masavoid by i7 i8 i9;  
      masapp by i10 i11 i12;  
      [i1 – i12];  
output: sampstat residual stand(stdyx) modindices;
```

Each *Mplus* command begins with a keyword followed by a colon. Each command must end with a semi-colon.

The **title** command specifies a title that will be printed at the top of each page of output. It is not required, but should be included for easy identification of different sets of output.

The **data** command is required and specifies the name and location of the data file. In this example I do not include a directory specification for the file because the data are in the same location as the *Mplus* program. If this is not the case, the name of the data file should be preceded by the directory in which the file resides.

Data in the goal orientation file are tab-delimited. Data that are tab-delimited or in which variables are separated by at least one space do not require a format statement. I include the statement

format is free, although this is the default specification, so if data are separate by spaces or tabs, this statement is not required.

The **variables** command specifies the names of the variables in the data file. Variable names should be eight characters or fewer. The variable names must be in the same order as the variables appear in the data file. The **missing** subcommand specifies that values of 0,8,and 9 are codes for missing values for all the variables (indicated by the keyword **all**). Inclusion of the **missing** subcommand automatically invokes full information maximum likelihood (FIML) missing data handling for CFA models.

The **model** command specifies the model to be analyzed. The keyword **by** means “measured by” and is used to specify a confirmatory factor analysis model in which four latent variables (“perfapp,” “peravoid,” “masavoid,” and “masapp”) are each measured by three observed variables. Here, the names of the latent variables are specified by the researcher and must follow the same length conventions as other variables. The names of the observed variables come after the keyword **by**.

The specification **[i1-i12]** indicates that the mean, or intercept, of each observed variable (i1 - i12) should be estimated.

By default, latent factors in CFAs are correlated and factor variances as well as each variable’s residual variance are estimated.

Also by default, *Mplus* sets the first loading for each factor to 1.0 to identify the model (in our example, the loadings for i1, i4, i7, and i10 would be set to 1.0).

In some cases, researchers prefer to identify the model (see Chapter 13 for more information on identification) by setting the factor variances to 1.0. To do this, replace the **model** syntax with the syntax below:

```
perfapp by i1* i2 i3;  
peravoid by i4* i5 i6;  
masavoid by i7* i8 i9;  
masapp by i10* i11 i12;  
perfapp@1 peravoid@1 masavoid@1 masapp@1;
```

The asterisks after i1, i4, i7, and i10 indicate that the loadings for these variables should be estimated rather than fixed to 1.0

The last command sets the variances of the four factors to 1.0 with the specification “@1.”

Finally, the **output** line requests the sample statistics (**sampstat**), the model residuals (**residual**), the standardized solution (**stand(stdyx)**), and modification indices (**modindices**).

The sample statistics include the variable means, variances, skewness and kurtosis values of the observed variables as well as their covariance and correlation matrices.

The model residuals are the differences between the observed and model-implied covariances. *Mplus* provides several different standardized solutions. The specification **stdyx** requests the completely standardized solution, in which both the latent and observed variables are standardized.

The modification indices are estimates of the amount by which the chi-square value would decrease if the associated parameter were to be included in the model.

See pages 383-388 in the text for more information on residuals and modification indices.

CFA Output

In this section I present selected output from the analysis.

Information on model fit is presented first. Here I show only the indices discussed in the text (see pages 376-381 for more information).

MODEL FIT INFORMATION

Chi-Square Test of Model Fit

| | |
|--------------------|---------|
| Value | 283.977 |
| Degrees of Freedom | 48 |
| P-Value | 0.0000 |

RMSEA (Root Mean Square Error Of Approximation)

| | | |
|--------------------------|-------|-------|
| Estimate | 0.069 | |
| 90 Percent C.I. | 0.062 | 0.077 |
| Probability RMSEA <= .05 | 0.000 | |

CFI/TLI

| | |
|-----|-------|
| CFI | 0.946 |
| TLI | 0.926 |

SRMR (Standardized Root Mean Square Residual)

| | |
|-------|-------|
| Value | 0.046 |
|-------|-------|

These values indicate some model misspecification.

The chi-square is a test of whether the observed covariance matrix is equal to the model-implied matrix, or, more broadly, whether the model fits the data. Rejection of the chi-square test indicates that the model does not fit the data.

A common cut-off value for the RMSEA is .06, with higher values indicating an unacceptable lack of fit.

The CFI compares the fit of the model to that of a null, or baseline model, and is a measure of the degree to which the model fits better than a model in which the variables are mutually uncorrelated. A common cut-off value is .95.

Finally, the SRMR is a function of the average of the residuals (observed – model-implied). Common cut-off values are .08-.09.

The model parameter estimates are presented next. Here, I present the unstandardized results. The completely standardized are shown in Figure 13.4 in the text (p. 373).

MODEL RESULTS

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|-------------|----------|-------|-----------|-----------------------|
| PERFAPP BY | | | | |
| I1 | 1.000 | 0.000 | 999.000 | 999.000 |
| I2 | 1.035 | 0.036 | 28.865 | 0.000 |
| I3 | 1.052 | 0.036 | 29.396 | 0.000 |
| PERAVOID BY | | | | |
| I4 | 1.000 | 0.000 | 999.000 | 999.000 |
| I5 | 0.772 | 0.060 | 12.807 | 0.000 |
| I6 | 1.195 | 0.081 | 14.837 | 0.000 |
| MASAVOID BY | | | | |
| I7 | 1.000 | 0.000 | 999.000 | 999.000 |
| I8 | 1.956 | 0.150 | 13.001 | 0.000 |
| I9 | 1.845 | 0.139 | 13.270 | 0.000 |
| MASAPP BY | | | | |
| I10 | 1.000 | 0.000 | 999.000 | 999.000 |
| I11 | 0.974 | 0.053 | 18.472 | 0.000 |
| I12 | 1.041 | 0.058 | 18.031 | 0.000 |

The values in the “estimate” column above are the factor loadings or coefficients. These should be large in magnitude and in the expected direction.

The standard errors are shown in the next column (labeled “S.E.”).

The third column contains the z-values, which test the hypothesis that the parameter estimate is equal to zero.

Finally, the two-tailed *p*-value for the z-tests are shown in the last column.

| | | | | |
|--------------------------|-------|-------|--------|-------|
| PERAVOID WITH PERFAPP | 0.751 | 0.071 | 10.612 | 0.000 |
| MASAVOID WITH PERFAPP | 0.197 | 0.034 | 5.837 | 0.000 |
| PERAVOID | 0.378 | 0.046 | 8.171 | 0.000 |
| MASAPP WITH PERFAPP | 0.308 | 0.043 | 7.234 | 0.000 |
| PERAVOID | 0.080 | 0.039 | 2.063 | 0.039 |
| MASAVOID | 0.159 | 0.025 | 6.477 | 0.000 |

The values above are the covariances among the four factors. The standardized solution provides the factor correlations.

| | | | | |
|-----------|-------|-------|--------|-------|
| Variances | | | | |
| PERFAPP | 1.575 | 0.105 | 14.968 | 0.000 |
| PERAVOID | 1.310 | 0.148 | 8.857 | 0.000 |
| MASAVOID | 0.432 | 0.062 | 7.010 | 0.000 |
| MASAPP | 0.663 | 0.060 | 11.037 | 0.000 |

The values above are the variances of the latent variables, or factors.

Below these are the residual, or unexplained, variance in the observed variables.

| | | | | |
|--------------------|-------|-------|--------|-------|
| Residual Variances | | | | |
| I1 | 0.816 | 0.050 | 16.358 | 0.000 |
| I2 | 0.649 | 0.048 | 13.608 | 0.000 |
| I3 | 0.671 | 0.049 | 13.739 | 0.000 |
| I4 | 2.445 | 0.133 | 18.389 | 0.000 |
| I5 | 2.038 | 0.107 | 18.984 | 0.000 |
| I6 | 1.071 | 0.111 | 9.639 | 0.000 |
| I7 | 1.594 | 0.076 | 21.104 | 0.000 |
| I8 | 0.609 | 0.078 | 7.815 | 0.000 |
| I9 | 1.104 | 0.081 | 13.701 | 0.000 |
| I10 | 0.725 | 0.044 | 16.658 | 0.000 |
| I11 | 0.327 | 0.030 | 10.867 | 0.000 |
| I12 | 0.713 | 0.044 | 16.204 | 0.000 |

The R^2 values below are presented at the end of the standardized solution and measure the proportion of variance in each observed variable that is explained by the latent variables. High values indicate that the observed variables are strongly related to the latent variables. From the values below we can see that I5 and I7 have low R^2 values, indicating they are not strongly related to their respective factors.

R-SQUARE

| Observed Variable | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|----------------------|----------|-------|-----------|-----------------------|
| I1 | 0.659 | 0.023 | 29.228 | 0.000 |
| I2 | 0.722 | 0.022 | 32.830 | 0.000 |
| I3 | 0.722 | 0.022 | 33.094 | 0.000 |
| I4 | 0.349 | 0.032 | 11.015 | 0.000 |
| I5 | 0.277 | 0.032 | 8.786 | 0.000 |
| I6 | 0.636 | 0.039 | 16.429 | 0.000 |
| I7 | 0.213 | 0.026 | 8.082 | 0.000 |
| I8 | 0.731 | 0.035 | 20.637 | 0.000 |
| I9 | 0.571 | 0.032 | 17.787 | 0.000 |
| I10 | 0.478 | 0.031 | 15.417 | 0.000 |
| I11 | 0.658 | 0.033 | 20.167 | 0.000 |
| I12 | 0.502 | 0.031 | 16.295 | 0.000 |

The standardized residuals are shown below. These are in a z-score metric, so values greater than |2.0| are typically taken as indications of misfit. Selected values are shown in Table 13.6 in the text, along with a discussion of possible reasons for the model misfit.

As can be seen from the table, many of the residuals are greater than |2.0|. This supports our earlier judgment of model misfit based on the fit indices.

Standardized Residuals (z-scores) for Covariances

| | I1 | I2 | I3 | I4 | I5 |
|-----|--------|---------|--------|--------|--------|
| I1 | 0.000 | | | | |
| I2 | 0.004 | 0.000 | | | |
| I3 | 4.252 | -22.306 | 0.000 | | |
| I4 | -3.995 | -0.840 | -3.669 | 0.000 | |
| I5 | -2.717 | 0.671 | -2.615 | 4.495 | 0.000 |
| I6 | -3.574 | 5.594 | 5.079 | 3.018 | -5.125 |
| I7 | -4.722 | -0.951 | -2.709 | 3.291 | 7.180 |
| I8 | -1.384 | 2.611 | 2.936 | -1.860 | 2.656 |
| I9 | -3.296 | -0.409 | 0.528 | -5.034 | 2.424 |
| I10 | 4.900 | 3.666 | 2.219 | 0.151 | 1.270 |
| I11 | -0.920 | -0.324 | -1.308 | -1.214 | -1.717 |
| I12 | -2.898 | -1.355 | -2.667 | -0.277 | -0.417 |

Standardized Residuals (z-scores) for Covariances

| | I6 | I7 | I8 | I9 | I10 |
|-----|---------|--------|---------|--------|--------|
| I6 | 999.000 | | | | |
| I7 | 2.189 | 0.000 | | | |
| I8 | -1.638 | -1.224 | 0.000 | | |
| I9 | -0.090 | -1.268 | 999.000 | 0.000 | |
| I10 | 0.566 | -2.759 | -1.173 | -1.180 | 0.000 |
| I11 | -0.266 | -2.241 | -0.284 | -0.245 | -0.711 |
| I12 | 1.696 | 2.244 | 2.114 | 2.076 | -1.025 |

| Standardized Residuals (z-scores) for Covariances | | |
|---|-------|-------|
| | I11 | I12 |
| I11 | 0.000 | |
| I12 | 4.701 | 0.000 |

The model modification indices are presented last and are shown below. By default, *Mplus* prints values greater than 10.0. To obtain all the modification indices, use the syntax **modindices (all)**.

The first column refers to the parameter to be added to the model. In this column “BY Statements” refer to loadings. For example, “PERFAPP BY I4” refers to the addition of the loading of I4 on PERFAPP. “WITH Statements” refer to covariances between pairs of residual variances. For example, “I3 WITH I1” refers to the covariance between the residuals of I3 and I1.

The second column, labeled “M.I.” refers to the value by which the chi-square value would decrease if the parameter in column 1 were to be added to the model (the modification index).

In the third column, the “E.P.C.” or Expected Parameter Change, is the estimated value of the parameter in question, if that parameter were to be added to the model.

The “Std E.P.C.” and the StdYX E.P.C.” are standardized versions of the EPC. In the “Std” version, the EPC is standardized based on the variances of the latent variables only, whereas in the “StdYX” version the EPC is standardized using the variances of both the latent and observed variables.

For example, the M.I. of 12.911 for “PERFAPP BY I4” indicates that, if I4 were allowed to load on PERFAPP the overall chi-square value would decrease by 12.911. The estimated value of the new loading would be -.229 in the unstandardized solution, -.287 in the Std standardized solution, and -.148 in the completely standardized (StdYX) solution.

MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index 10.000

| | M.I. | E.P.C. | Std E.P.C. | StdYX E.P.C. |
|----------------|--------|--------|------------|--------------|
| BY Statements | | | | |
| PERFAPP BY I4 | 12.911 | -0.229 | -0.287 | -0.148 |
| PERFAPP BY I6 | 22.757 | 0.326 | 0.409 | 0.239 |
| PERFAPP BY I10 | 20.215 | 0.126 | 0.158 | 0.135 |
| PERAVOID BY I1 | 30.608 | -0.235 | -0.269 | -0.174 |
| PERAVOID BY I2 | 15.397 | 0.161 | 0.185 | 0.121 |
| PERAVOID BY I7 | 13.124 | 0.185 | 0.212 | 0.149 |
| MASAVOID BY I1 | 16.106 | -0.234 | -0.154 | -0.099 |
| MASAVOID BY I5 | 15.097 | 0.406 | 0.267 | 0.159 |

WITH Statements

| | | | | | |
|-----|---------|--------|--------|--------|--------|
| I3 | WITH I1 | 17.588 | 0.311 | 0.311 | 0.420 |
| I3 | WITH I2 | 19.874 | -0.363 | -0.363 | -0.551 |
| I4 | WITH I3 | 10.563 | -0.170 | -0.170 | -0.132 |
| I5 | WITH I4 | 17.395 | 0.378 | 0.378 | 0.169 |
| I6 | WITH I1 | 12.569 | -0.157 | -0.157 | -0.168 |
| I6 | WITH I3 | 15.912 | 0.172 | 0.172 | 0.203 |
| I6 | WITH I5 | 33.827 | -0.591 | -0.591 | -0.400 |
| I7 | WITH I1 | 13.436 | -0.153 | -0.153 | -0.134 |
| I7 | WITH I4 | 11.602 | 0.231 | 0.231 | 0.117 |
| I7 | WITH I5 | 40.375 | 0.386 | 0.386 | 0.214 |
| I9 | WITH I4 | 23.206 | -0.310 | -0.310 | -0.189 |
| I9 | WITH I8 | 12.798 | 0.650 | 0.650 | 0.793 |
| I10 | WITH I1 | 19.295 | 0.133 | 0.133 | 0.174 |
| I12 | WITH I7 | 16.777 | 0.157 | 0.157 | 0.147 |