**Testing for Tau-equivalence and Parallelism Using M*plus***

In this document I explain how to use M*plus* to obtain models based on the assumptions of tau-equivalence, essential tau-equivalence (true-score equivalence), and parallelism. Tests of these assumptions can be obtained by comparing the chi-square values of the models using chi-square difference tests, as explained in the text (pp. 394-395).

Note that the usual CFA model is based on a congeneric model in which item loadings, means, and measurement error variances are all allowed to vary across items. Because of this, no additional model specifications are needed to obtain the congeneric model. Values for that model can be obtained from the document “Confirmatory Factor Analysis Using M*plus*.”

The data from this study are the same as those used in the CFA document and consist responses of 1022 people to items on the Achievement Goal Questionnaire (Elliot & McGregor, 2001) collected as part of a study by Finney, Pieper, and Barron (2004). There are no missing data, but I have included the “missing” command used to specify missing data in *Mplus* to demonstrate how this would be done.

The response scale ranged from 1 = not at all true of me to 7 = very true of me. The data are in tab-delimited format and can be found in the file “goal orientation.dat”

**Tau-equivalent Model Syntax**

The syntax is based on that for the confirmatory factor analysis and repeats the commands for that analysis. I include all commands below but only comment on those that are new to these analyses.

M*plus* commands are in boldface. Non-boldface words indicate information that must be provided by the researcher for their specific data.

I first show the syntax and results for the tau-equivalent model and then show those for the essentially tau-equivalent and parallel models.

**Tau-equivalent Model**

**Title:** specification of tau-equivalent and parallel models for goal orientation data;

**Data**: **file is** goal2.dat;

**format is** free;

**Variable**: **names are** i1 i2 i3 i4 i5 i6 i7 i8 i9 i10 i11 i12;

**missing all** (8,9,10);

**usevariables =** i7 i8 i9;

*! The first set of commands below specifies that item loadings must be equal within factors. The numbers in parentheses at the end of each “by” statement indicate that the values of all parameter estimates specified by that statement must be equal in value. For example, the statement “perfapp* ***by*** *i1\* i2 i3 (1);” indicates that the loadings for i1, i2, and i3 should be equal. Using a different number for each set of “by” statements (here, 1, 2, 3, 4) is indicates that the loadings for each “by” statement must be equal, but that the sets of loadings can differ in value across “by” statements. For example, loading estimates for i1, i2, and i3 must be equal, and estimates for i4, i5, and i6 must be equal, but estimates can differ across the two sets.*

*! Inclusion of asterisks (\*) after the names of the first item in each line indicate that the item loading should be estimated. Recall that the default in Mplus is to set the loading of the first indicator to 1.0. Use of the asterisk overrides this default.*

**Model:** perfapp **by** i1\* i2 i3 (1);

peravoid **by** i4\* i5 i6 (2);

masavoid **by** i7\* i8 i9 (3);

masapp **by** i10\* i11 i12 (4);

*! The next set of commands specifies that item means must be equal within factors. Note that each set of item names is enclosed within brackets [ ]. In Mplus, enclosing item names in brackets indicates that item means (or intercepts) should be estimated. Inclusion of the sets of commands for both equality of item loadings and item means results in a tau-equivalent model. As with the loading commands above, the numbers in parentheses are used to specify that estimates of item means must be equal within each factor.*

[i1 i2 i3] (5);

[i4 i5 i6] (6);

[i7 i8 i9] (7);

[i10 i11 i12] (8);

*! The last set of commands below specifies that items’ measurement error variances must be equal within factors. The exclamation points (!) before each command causes Mplus to ignore these, as equality of measurement error variances is not assumed for tau-equivalent models As before, the numbers in parentheses following each command are used to indicate that estimates for each set of measurement error variances must be equal.*

! i1 i2 i3 (9);

! i4 i5 i6 (10);

! i7 i8 i9 (11);

! i10 i11 i12 (12);

perfapp@1; peravoid@1; masavoid@1; masapp@1;

! *The command line above sets the variance of each factor equal to 1.0. This specification must be included to identify the model because all loadings are estimated.*

*The italicized lines in the syntax above are comments that explain how different models are obtained in Mplus. Comments can be included by preceding a line with an explanation mark (!). Any syntax after the explanation mark will appear in green within Mplus and will not be read by the program. This feature is useful in analyses such as these, in which the different models are invoked by the inclusion or exclusion of particular lines in the syntax.*

To obtain results from a tau-equivalent model, I included the commands for equality of item loadings and of item means shown above. I commented out (using explanation points) the syntax for equality of the items’ measurement error variances.

These specifications resulted in the output below. I have omitted the factor correlations as they are not relevant to these analyses.

Chi-Square Test of Model Fit

Value 1371.051

Degrees of Freedom 64

P-Value 0.0000

MODEL RESULTS

Two-Tailed

Estimate S.E. Est./S.E. P-Value

PERFAPP BY

I1 1.289 0.033 39.257 0.000

I2 1.289 0.033 39.257 0.000

I3 1.289 0.033 39.257 0.000

PERAVOID BY

I4 1.114 0.039 28.718 0.000

I5 1.114 0.039 28.718 0.000

I6 1.114 0.039 28.718 0.000

MASAVOID BY

I7 1.109 0.037 30.090 0.000

I8 1.109 0.037 30.090 0.000

I9 1.109 0.037 30.090 0.000

MASAPP BY

I10 0.797 0.023 33.995 0.000

I11 0.797 0.023 33.995 0.000

I12 0.797 0.023 33.995 0.000

As can be seen from the output above, the loading values have been set to equality with each factor, as specified in the syntax.

Intercepts

I1 5.094 0.043 117.210 0.000

I2 5.094 0.043 117.210 0.000

I3 5.094 0.043 117.210 0.000

I4 4.546 0.044 102.854 0.000

I5 4.546 0.044 102.854 0.000

I6 4.546 0.044 102.854 0.000

I7 4.221 0.042 99.906 0.000

I8 4.221 0.042 99.906 0.000

I9 4.221 0.042 99.906 0.000

I10 6.015 0.030 201.070 0.000

I11 6.015 0.030 201.070 0.000

I12 6.015 0.030 201.070 0.000

**The intercept values above are the item means. These have also been set to equality with each factor, as specified in the syntax.**

Variances

PERFAPP 1.000 0.000 999.000 999.000

PERAVOID 1.000 0.000 999.000 999.000

MASAVOID 1.000 0.000 999.000 999.000

MASAPP 1.000 0.000 999.000 999.000

**The factor variances have been set to 1.0 to identify the model.**

Residual Variances

I1 0.787 0.048 16.348 0.000

I2 0.665 0.044 15.091 0.000

I3 0.755 0.047 16.001 0.000

I4 2.572 0.136 18.885 0.000

I5 1.824 0.102 17.801 0.000

I6 1.656 0.098 16.888 0.000

I7 2.483 0.135 18.336 0.000

I8 0.821 0.064 12.762 0.000

I9 1.315 0.081 16.333 0.000

I10 0.812 0.045 17.998 0.000

I11 0.398 0.030 13.371 0.000

I12 0.743 0.042 17.841 0.000

**For the tau-equivalent model, the measurement error variances above are not required to be equal. As can be seen from the values above, they are not.**

**Essentially Tau-equivalent (true-score) Model**

The essentially tau-equivalent model is obtained by dropping the requirement that item means must be equal. You can do this in two ways. The first is to simply omit the commands for item means. Because the default in M*plus* is to include item means and to allow them to vary across items, there is no real need to include these commands unless you want to constrain them in some way.

The other way to do this is to remove the numbers in parentheses (5, 6, 7, and 8) from the item mean commands in the previous tau-equivalent model syntax. This will reduce the commands for item means to:

[i1 i2 i3];

[i4 i5 i6];

[i7 i8 i9];

[i10 i11 i12];

All other commands remain the same as those for the tau-equivalent model.

Making the change for item means results in the output below:

Chi-Square Test of Model Fit

Value 440.638

Degrees of Freedom 56

P-Value 0.0000

MODEL RESULTS

Two-Tailed

Estimate S.E. Est./S.E. P-Value

PERFAPP BY

I1 1.293 0.033 39.456 0.000

I2 1.293 0.033 39.456 0.000

I3 1.293 0.033 39.456 0.000

PERAVOID BY

I4 1.139 0.038 29.814 0.000

I5 1.139 0.038 29.814 0.000

I6 1.139 0.038 29.814 0.000

MASAVOID BY

I7 1.067 0.034 31.212 0.000

I8 1.067 0.034 31.212 0.000

I9 1.067 0.034 31.212 0.000

MASAPP BY

I10 0.813 0.023 35.449 0.000

I11 0.813 0.023 35.449 0.000

I12 0.813 0.023 35.449 0.000

PERAVOID WITH

PERFAPP 0.504 0.033 15.124 0.000

MASAVOID WITH

PERFAPP 0.219 0.037 5.868 0.000

PERAVOID 0.574 0.035 16.304 0.000

MASAPP WITH

PERFAPP 0.301 0.035 8.655 0.000

PERAVOID 0.076 0.042 1.794 0.073

MASAVOID 0.293 0.038 7.690 0.000

Intercepts

I1 5.155 0.049 105.074 0.000

I2 5.201 0.048 109.117 0.000

I3 4.915 0.048 102.108 0.000

I4 4.860 0.060 80.545 0.000

I5 4.713 0.055 85.676 0.000

I6 4.192 0.052 80.937 0.000

I7 5.103 0.050 101.127 0.000

I8 4.135 0.045 92.379 0.000

I9 3.890 0.048 80.711 0.000

I10 5.756 0.037 155.984 0.000

I11 6.209 0.031 201.538 0.000

I12 5.889 0.037 159.474 0.000

Variances

PERFAPP 1.000 0.000 999.000 999.000

PERAVOID 1.000 0.000 999.000 999.000

MASAVOID 1.000 0.000 999.000 999.000

MASAPP 1.000 0.000 999.000 999.000

Residual Variances

I1 0.788 0.048 16.539 0.000

I2 0.650 0.043 15.175 0.000

I3 0.697 0.044 15.745 0.000

I4 2.424 0.128 18.941 0.000

I5 1.796 0.099 18.047 0.000

I6 1.444 0.087 16.608 0.000

I7 1.463 0.083 17.522 0.000

I8 0.908 0.062 14.556 0.000

I9 1.235 0.074 16.734 0.000

I10 0.732 0.040 18.278 0.000

I11 0.310 0.025 12.578 0.000

I12 0.734 0.040 18.382 0.000

**As can be seen from the parameter estimates above, item loadings and measurement error variances, but not item means (intercepts) are now held equal within each factor. Removal of the constraints on item means resulted in a gain of 8 degrees of freedom (2 for each factor) and a decrease of the chi-square value to 440.638.**

**Parallel Model**

Syntax for the parallel model is the same as that for the tau-equivalent model *except* that the exclamation points before the commands for measurement error variances are removed, thus forcing these parameters to be equal within each factor. To obtain the parallel model, the four command lines below are thus added to the syntax for the tau-equivalent model.

i1 i2 i3 (9);

i4 i5 i6 (10);

i7 i8 i9 (11);

i10 i11 i12 (12);

Adding in these commands results in the output below:

Chi-Square Test of Model Fit

Value 1582.008

Degrees of Freedom 72

P-Value 0.0000

MODEL RESULTS

Two-Tailed

Estimate S.E. Est./S.E. P-Value

PERFAPP BY

I1 1.289 0.033 39.245 0.000

I2 1.289 0.033 39.245 0.000

I3 1.289 0.033 39.245 0.000

PERAVOID BY

I4 1.126 0.039 28.853 0.000

I5 1.126 0.039 28.853 0.000

I6 1.126 0.039 28.853 0.000

MASAVOID BY

I7 0.957 0.035 27.357 0.000

I8 0.957 0.035 27.357 0.000

I9 0.957 0.035 27.357 0.000

MASAPP BY

I10 0.806 0.024 33.449 0.000

I11 0.806 0.024 33.449 0.000

I12 0.806 0.024 33.449 0.000

Intercepts

I1 5.090 0.043 117.857 0.000

I2 5.090 0.043 117.857 0.000

I3 5.090 0.043 117.857 0.000

I4 4.588 0.043 105.605 0.000

I5 4.588 0.043 105.605 0.000

I6 4.588 0.043 105.605 0.000

I7 4.376 0.038 115.602 0.000

I8 4.376 0.038 115.602 0.000

I9 4.376 0.038 115.602 0.000

I10 5.952 0.029 204.652 0.000

I11 5.952 0.029 204.652 0.000

I12 5.952 0.029 204.652 0.000

Variances

PERFAPP 1.000 0.000 999.000 999.000

PERAVOID 1.000 0.000 999.000 999.000

MASAVOID 1.000 0.000 999.000 999.000

MASAPP 1.000 0.000 999.000 999.000

Residual Variances

I1 0.734 0.023 31.968 0.000

I2 0.734 0.023 31.968 0.000

I3 0.734 0.023 31.968 0.000

I4 1.987 0.062 31.970 0.000

I5 1.987 0.062 31.970 0.000

I6 1.987 0.062 31.970 0.000

I7 1.644 0.051 31.969 0.000

I8 1.644 0.051 31.969 0.000

I9 1.644 0.051 31.969 0.000

I10 0.645 0.020 31.969 0.000

I11 0.645 0.020 31.969 0.000

I12 0.645 0.020 31.969 0.000

**As can be seen from the parameter estimates above, item loadings, means, and measurement error variances are now held equal within each factor. The additional constraints on the measurement error variances resulted in an increase of the chi-square value to 1582.008.**

**The significance of the chi-square increases obtained from comparing the congeneric to the essentially tau-equivalent model, the essentially tau-equivalent to tau-equivalent model, and the tau-equivalent to the parallel model are typically summarized in a table such as that shown below. (Results for the congeneric model were obtained from the original CFA output).**

|  |  |  |  |
| --- | --- | --- | --- |
| ***Model*** | ***χ2*** | ***df*** | ***Δ χ2 (df)*** |
| *Congeneric* | 283.98 | 48 |  |
| *Essentially tau-equivalent* | 440.64 | 56 | 156.66\* (8) |
| *Tau-equivalent* | 1371.05 | 64 | 930.41\* (8) |
| *Parallel* | 1582.01 | 72 | 210.96\* (8) |

**Addition of the constraints imposed by the essentially tau-equivalent model result in a statistically significant increase of 156.66, indicating that the data deviate significantly from essential tau-equivalence. Alternatively, we could interpret this as meaning that the item loadings within a factor are not equal. Addition of the constraint imposed by the tau-equivalent model that item means must be equal results in a chi-square increase of 930.41.Thus, the Imposition of constraints on the item means yields a statistically significant decrement in fit, indicating that the item means are not equal within a factor.**

**Finally, addition of the constraints imposed by the parallel model to those imposed by the tau-equivalent model also results in a statistically significant increase in chi-square (210.96 with 8 degrees of freedom). This indicates that the data deviate significantly from the parallel model, which is not surprising because parallelism cannot hold if tau-equivalence does not. This is also a test of the equality of measurement error variances, and indicates that these are not equal across sets of items.**