CHAPTER 1

Mathematics Interventions

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LEARNING OUTCOMES

After studying this chapter, you will be able to answer the following questions:

1. What are the Common Core State Standards for Mathematics?
2. Who are students with mathematics difficulties and mathematics learning disabilities?
3. What are multi-tiered systems of support and response to intervention?
4. How can mathematics interventions be intensified in Tiers 2 and 3 for students with mathematics difficulties and mathematics learning disabilities?

Mathematics is an important part of the curriculum for preschool through 12th grade and extends into postsecondary education as a means for preparing students to be competitive in today's workforce. Not only is mathematics knowledge important for work in many professions, but we know that the ability to reason mathematically and apply mathematics knowledge is critical for students to address daily living tasks that we all encounter. In fact, mathematics is such an important part of the curriculum and everyday living that professional organizations have provided information to help us better understand what mathematical literacy means in today's society.

Mathematical literacy is a term that describes the ability to reason and communicate about mathematics to solve problems in the classroom and everyday life (National Mathematics Advisory Panel [NMAP], 2008). Moreover, mathematical literacy refers to the ability to formulate, apply, and translate mathematical understandings in different situations including everyday life using mathematical concepts, procedures, and tools (Organization for Economic Co-operation and Development [OECD], 2012). The National Council of Teachers of Mathematics (NCTM, 2000) stressed the urgency in better preparing our children and adolescents for the "real world," where the translation of mathematical literacy abilities is critical. NCTM noted, "The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase" (p. 4); "those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. A lack of mathematical competence keeps those doors closed" (p. 5).

Consider for a moment how and when you use your mathematics knowledge in everyday life. What ideas did you identify where you use mathematics to help? If you identified the use of mathematics for managing your finances, purchasing household items, paying taxes, computing a tip, cooking with recipes, and using different types of measurement, as examples, then it is clear we all need a solid understanding in mathematical literacy to navigate school, work, and daily living.

Even in our technologically advanced society that will only continue to provide ways to use technology in schools, work, and home, we must still understand the mathematics to be sure our solutions to problems are reasonable mathematically. For instance, when using a calculator to compute a tip, you need to know how to enter numbers into the tool and the operation to select to calculate the tip; this sounds simple to do. But translating the tip percentage into a decimal equivalence and clicking the correct operation are necessary prerequisite skills to easily calculate the tip. Or perhaps you prefer to round the total bill and then compute the tip percentage. Even if the bill has percentage options, such as a 15% or 20% tip, you still have to use addition to calculate the final amount.

Just this one task alone, computing the tip on a restaurant bill, involves various mathematics abilities, illustrating how important mathematics literacy is for all of us. For students with mathematics difficulties, being mathematically literate is not easy because of the problems they experience with understanding mathematics concepts and using mathematics procedures to solve problems.

We can see that a focus on the mathematics abilities and challenges of all students is paramount for educators to ensure children and adolescents are prepared to meet future demands of the workforce and daily living. Unfortunately, consider the following results from international and national mathematics assessments. At the international level, the Program for International Student Assessment (PISA; OECD, 2012) is a measure given every 3 years to 15-year-old students across multiple countries to assess their application of mathematics to real-world problems. U.S. students' average score was 481 on a scale of 0–1,000; the 2012 average score showed U.S. students ranking 31st out of 34 OECD countries and 31 partner countries (OECD, 2012).

At the national level, on the National Assessment of Educational Progress (NAEP), average grade 4 mathematics scores for students with disabilities showed a decrease of 4 points compared to the 2015 findings. For grade 8, results showed no significant score change from the 2015 findings. For students with and without disabilities, results indicated that on average, only 49% of fourth graders with disabilities compared to 84% of students without disabilities scored at or above the *Basic* level (National Center for Education Statistics [NCES], 2017). In eighth grade, average scores of students with and without disabilities were even more alarming with 31% and 75%, respectively, scoring at or above the *Basic* level.

Taken together, it is evident that, in general, U.S. students' mathematics performance is disconcerting compared to other industrialized countries and U.S. students with disabilities are at a decided disadvantage compared to their peers without disabilities. It is easy to conclude that the development and application of mathematical literacy at school and in daily living are important for all students, including those students with mathematics difficulties (MD) and mathematics learning disabilities (MLD). But as can be seen in the international and national assessment results, gaps in our students' mathematical knowledge and performance are evident. These gaps are not acceptable and cause major concern among educators and policy makers. Therefore, educators must be equipped to address the issues students with MD and MLD show in their classroom mathematics activities related to their district's and state's standards and assessments. Specifically, students with MD and MLD must receive intensive mathematics interventions in mathematical domains if they are to stand a chance of succeeding in school and beyond.

Thus, the purpose of this book is to provide readers with content in several mathematics areas that are deemed to be the most critical aspects of mathematics interventions for students with MD and MLD. Researchers who work in developing and validating mathematics interventions have contributed chapters to this book in areas that educators must teach to students with MD and MLD.

What will you read about in the chapters in this book? First, you will find descriptions and explanations of mathematics areas. You will also learn about evidence-based practices for teaching the various mathematics areas. Collectively, the chapters will include connections to the Common Core State Standards for Mathematics (CCSSM; National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010), information about MD and MLD as they relate to each chapter, and examples of how each chapter's content can be operationalized within a response-to-intervention (RTI) model that can be used to intensify mathematics interventions for struggling students. Misconceptions and vocabulary associated with specific content knowledge is presented. Finally, in each chapter, you will find a reproducible of specific content that you can use in your classroom to plan interventions. Now, take a moment to review the table of contents for this book to find out which mathematics areas are most critical for struggling students to learn. Note that the last chapter, "Use of Technology for Intensifying Mathematics Intervention," contains information that can be used with the mathematics areas in the other chapters.

To get started, in this chapter, as a foundation for reading the remaining chapters in this book, we provide information about the characteristics of students with mathematics difficulties. Let's begin with learning more about the CCSSM as the framework for content in this book.

Common Core State Standards for Mathematics

In 2010, the NGA and CCSSO published the CCSSM. Aware of how U.S. students were performing commensurate to their U.S. and international peers on national (e.g., NAEP) and international (e.g., PISA, Trends in International Mathematics and Science Study [TIMSS]) assessments, the CCSSM authors recognized that a more "focused and coherent" mathematics curriculum in U.S. schools could be informed with Standards that tap the most critical concepts and skills needed at each grade level. Well over half of the states and the District of Columbia have adopted the CCSSM; the remaining states that have not adopted the CCSSM have their own standards or have implemented an adapted version.

So, what are the CCSSM? The CCSSM are evidence based and incorporate "learning progressions" or "learning trajectories" of important mathematics domains and topics. A learning progression or learning trajectory is evidence based regarding the typical developmental progression of mathematical concepts and skills across the grades, representing what we know about how mathematics concepts and skills build on each grade level's content (Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). Learning trajectories are empirically supported "descriptions of children's thinking as they learn to achieve specific goals in a mathematical domain" (Sarama & Clements, 2009, p. ix). The CCSSM provide guidance about what students should know and be able to apply at the end of each grade; that is, the CCSSM offer grade-level expectations. The CCSSM also were designed with the idea that upon graduating from high school, students would possess important concepts and skills needed for postsecondary education, work requirements, and life activities. Today, we think about the CCSSM as being "college readiness" standards.

Take a moment to locate the CCSSM at *www.corestandards.org/wp-content/uploads/Math_Standards1.pdf*. In examining the CCSSM, you will find two major areas. The first area is "Mathematical Content" across the grade levels. Notice that the standards consist of domains, standards, and clusters of related standards for mathematical content. As noted in the CCSSM, "Domains are larger groups of related standards, standards define what students should understand and be able to do, and clusters are groups of related standards" (NGA & CCSSO, 2010, p. 5). Let's look at an example from the fourth-grade CCSSM. One domain is called "Number and Operations—Fractions," and this domain has three standards, each with clusters of related standards. Look at the first standard, "Extend understanding of fraction equivalence and ordering" (p. 30). Notice that for this standard, fraction models are recommended to represent fraction equivalence and fraction comparisons. Both of these concepts, equivalence and comparisons, are critical for students to understand and be able to represent, because these concepts are two critical cornerstones for more advanced work with fractions.

The second area is "Mathematical Practice." Students are expected to understand Mathematical Content with opportunities to connect Mathematical Practice with the content knowledge. Mathematical Practice stems from the NCTM process standards and an important publication, *Adding It Up* (National Research Council, 2001), which focuses on adaptive reasoning, strategic competence, and conceptual understanding, procedural fluency, and productive disposition" (NGA & CCSSO, 2010, p. 6). The Mathematical Content and Mathematical Practice become the intersection for understanding, modeling, and reasoning about concepts and for developing proficiencies in mathematical fluency. Table 1.1 shows the CCSSM (NGA & CCSSO, 2010) Mathematical Content and Mathematical Practice.

TABLE 1.1. CCSSM Mathematical Content and Mathematical Practice for Kindergarten through High School

Mathematical Content	Mathematical Practice
• Counting and Cardinality • Operations and Algebraic Thinking • Number and Operations in Base Ten • Number and Operations—Fractions • Measurement and Data • Geometry • Ratios and Proportional Relationships • The Number System • Expressions and Equations • Statistics and Probability • Number and Quantity • Algebra • Functions • Modeling	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Note. From National Governors Association Center for Best Practices and Council of Chief State School Officers (2010). Copyright © 2010. All rights reserved.

Taken together, the CCSSM information can assist state and school district leaders in making decisions about next steps for identifying their mathematics curriculum, assessments, and instruction and helping students with MD and MLD improve their mathematics achievement. We know that, because of the hierarchical nature of mathematics meaning, later grade content knowledge is based on earlier grade knowledge, and that students with MD and MLD are disadvantaged because of the "holes" in their knowledge and understanding of CCSSM concepts and skills.

Students with MD and MLD

Information about students with MD and MLD frame the content of each chapter because it is for these students that we present our content. Let's find out more about some of the characteristics of students with MD and MLD. It is important to note that both groups of students will have similar learning characteristics associated with mathematics; however, students with MLD will exhibit more persistent, chronic mathematics difficulties throughout their lives due to the associated learning disability in mathematics.

MD refers to those children and adolescents who have learning problems that make it challenging for them to understand mathematics instruction. Although these students are not diagnosed with MLD, they often seem perplexed with simple mathematics areas, thus requiring more deliberate instruction on challenging areas. Sometimes, the MD group of students is referred to as low achievers, and they are usually identified as scoring between the 11th and 25th percentiles on mathematics assessments. For example, elementary-level students might struggle with understanding whole-number concepts, place value, and number combinations; secondary-level (middle and high school) students likely find ratios and proportional thinking and algebra difficult to understand. Even at the preschool and kindergarten levels, parents and teachers realize that some young children have problems with basic skills such as counting from 1 to 10, learning number names, and comparing one quantity to another quantity with counters to tell which is "more than," "less than," or if they are "the same." Thus, it is not surprising that students with MD demonstrate low achievement in mathematics with whole-number concepts in the elementary grades and rational numbers and algebra in later grades.

We also know that there is another group of students who have *developmental dyscalculia,* another name for MLD. This group of students typically scores at or below the 10th percentile on mathematics performance measures, which implies that these students have very low mathematics performance compared to their peers. Dyscalculia refers to difficulties in learning arithmetic, including problems such as understanding number, doing arithmetical calculations, and computing number combination facts in mathematics. Students may also have challenges with telling time on an analog clock, counting money and making change, learning and remembering procedures for solving problems, and identifying and knowing the meaning of symbols (Butterworth, 2010). Unfortunately, this pattern of chronic very low mathematics performance continues across the grade levels and into adulthood. Dyscalculia and MLD are often used interchangeably; in this book, we use the term MLD because it is more commonly used in schools. In Table 1.2, you can find a list of signs of dyscalculia at different ages (Understood, 2014–2019).

So, how pervasive are MD and MLD? Did you know that about 5–6% of the school-age population is diagnosed as having MLD in one or more mathematical areas? Also, about 10% of the school-age population is identified as having chronic mathematics difficulties, but these children are not diagnosed as having MLD. Thus, when looking at these percentages, we can see that a substantial number of children and adolescents have poor mathematics performance, which is persistent and pervasive across the grades and will likely extend into postsecondary mathematics courses and adulthood mathematics-related activities (Geary, 2011; Shalev, Manor, & Gross-Tsur, 2005; Swanson, 2006).

Additional information is available about the ramifications of students with MD and MLD experiencing chronic problems learning the mathematics curriculum. For example, research findings have shown that persistent mathematics learning problems contribute to a mathematics achievement performance gap between students with MD and MLD and their typically achieving peer group. Findings indicated that the achievement gap continues to widen as students fall further behind because these struggling students do not master important foundational and conceptual knowledge, which supports their mathematics learning in higher grades (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Morgan, Farkas, & Wu, 2011). Students with MD and MLD have difficulties with processing numerical relations (e.g., 53 > 49) quickly, retrieving (remembering) solutions for number combinations (i.e., basic facts), and solving word problems. These problems hinder students' ability to "catch up" to their typically achieving peers (Geary, Hoard, Nugent, & Bailey, 2012).

The good news is that, in recent years, much more research attention has focused on students who have MD and MLD in the earlier grades (Bryant et al., 2011; Clarke et al., 2014; Dyson, Jordan, & Glutting, 2013) and the secondary grades (Dougherty, Bryant, Bryant, & Shin, 2017; Krawec, 2014). Researchers are learning about the effects of carefully designed studies to determine what practices can be identified as making a difference with student understanding of and performance in mathematics. Researchers are studying instructional components and programs that teachers and interventionists can use with their students with MD and MLD, usually in special education or under the multi-tiered systems of support (MTSS) and RTI models. Now, let's turn our attention to a brief overview about MTSS and RTI, and how mathematics instruction can occur for struggling students, including those with MD and MLD.

TABLE 1.2. Signs of Dyscalculia at Different Ages

Signs of Dyscalculia in Preschool

- Has trouble learning to count and skips over numbers long after kids the same age can remember numbers in the right order.
- Doesn't seem to understand the meaning of counting. For example, when you ask for five blocks, she or he just hands you a large group of blocks, rather than counting them out.
- Struggles to recognize patterns, like smallest to largest or tallest to shortest.
- Has trouble understanding number symbols, like making the connection between "7" and the word *seven*.
- Struggles to connect a number to an object, such as knowing that "3" applies to groups of things like 3 cookies, 3 cars, or 3 kids.

Signs of Dyscalculia in Elementary School

- Has difficulty learning and recalling basic math facts, such as 2 + 4 = 6.
- Still uses fingers to count instead of using more advanced strategies (like mental math).
- Struggles to identify math signs like + and – and to use them correctly.
- Has a tough time understanding math phrases, like greater than and less than.
- Has trouble with place value, often putting numbers in the wrong column.

Signs of Dyscalculia in Middle School

- Struggles with math concepts like commutativity (3 + 5 is the same as 5 + 3) and inversion (being able to solve 3 + 26 – 26 without calculating).
- Has a tough time understanding math language and coming up with a plan to solve a math problem.
- Has trouble keeping score in sports games and gym activities.
- Has difficulty figuring out the total cost of things and often runs out of money on his or her lunch account.
- May avoid situations that require understanding numbers, like playing games that involve math.

Signs of Dyscalculia in High School

- Struggles to understand information on charts and graphs.
- Has trouble applying math concepts to money, such as making exact change and figuring out a tip.
- Has trouble measuring things like ingredients in a simple recipe or liquids in a bottle.
- Lacks confidence in activities that require understanding speed, distance, and directions, and may get lost easily.
- Has trouble finding different approaches to a math problem, such as adding the length and width of a rectangle and doubling the answer to solve for the perimeter (rather than adding all the sides).

Note. Adapted from Understood For All, Inc. (2014–2019).

MTSS and RTI Models

The content in our chapters is based on the idea that the MTSS and RTI models provide a way to think about how to intensify interventions for struggling students. We focus on three tiers of instruction, anticipating that you will see how instruction is intensified depending on the needs of students.

The MTSS and RTI models consist of evidence-based interventions and progress monitoring measures to provide important supports to students who have MD or MLD. The goal is to reduce inappropriate referrals to special education due to poor instruction and to improve student achievement. According to the Every Student Succeeds Act (2015), the term *MTSS* refers to "a comprehensive continuum of evidence-based, systemic practices to support a rapid response to students' needs, with regular observation to facilitate data-based instructional decision-making" (§ 7801[33]). MTSS models focus on preventing student difficulties and providing services within tiered and increasingly intensive levels of support (Kilgus & Eklund, 2016). MTSS is a broader framework including the RTI approach for differentiating instruction and intensifying interventions for students who, in this case, struggle with learning and remembering key mathematical content across the grades.

RTI models are based on screening for at-risk students; providing high-quality, research-based instruction in general core education; conducting progress monitoring; and providing multi-tiered intensive instruction to those in need (e.g., Memorandum to Chief State School Officers, 2008). RTI is usually characterized as having three tiers, Tier 1, Tier 2, and Tier 3, of prevention and intervention. Tier 1 is thought of as core or general education mathematics instruction, aligned with state or national standards for *all* students, including students with MD and MLD. In Tier 1, specific practices are recommended for elementary- and secondary-level mathematics core concepts and skills to ensure high-quality mathematics instruction for all students. Table 1.3 shows a list of recommended policies and practices that teachers should plan on incorporating into their daily mathematics instruction.

Tier 1 also features differentiated instruction for those students who are struggling in mathematics and includes, for example, providing adaptations to instruction through the use of different instructional materials and ways of presenting concepts. In Tier 2 (about 15% of school-age students) and in Tier 3 (about 5% of school-aged students), a universal screening process is used at the beginning of the year to identify these struggling students. Students may then be assigned to receive Tier 2 intervention, which is supplemental to core mathematics instruction, or to Tier 3, which focuses on more intensified intervention aimed at individual learning needs. Teachers and mathematics interventionists can provide intensified interventions to these groups of students to address their specific learning needs. For these smaller percentages of students, interventions in critical mathematical content occurs across the school year with smaller groups, adapted instruction, and frequent progress monitoring. Now we examine how teachers can intensify mathematics interventions to address the needs of their students with MD or MLD who are receiving Tier 2 or Tier 3 interventions.

TABLE 1.3. 10 Key Policies and Practices for Elementary and Secondary Mathematics

All students can become proficient in mathematics when:

1. Students are fluent with addition, subtraction, multiplication, and division number combinations. Students know the 390 mathematics facts.
2. Students master key algebraic-readiness concepts for fractions, decimals, integers, ratios and proportions, and expressions and equations.
3. Students learn effective problem-solving strategies for different types of word-problem structures. Teachers present "real life" word problems for students to solve on a daily basis.
4. Teachers differentiate mathematics instruction for diverse learners.
5. Teachers use explicit instruction. Teachers verbalize explanations of concepts and steps for solving problems.
6. Teachers provide multiple opportunities for practice and promote student engagement.
7. Students make their mathematics thinking visible by talking about their solution process, drawing a picture, or making a graph using mathematically correct vocabulary.
8. Teachers help students to solve mathematics problems using manipulatives and models to bridge concrete to symbolic understandings of mathematics.
9. Students are given solved problems (correctly and incorrectly solved with misconceptions) to discuss how the problems were solved.
10. Teachers collect data regularly to determine whether their students are benefiting from instruction and use the data to make instructional decisions for subsequent lessons.

Note. Adapted from the Meadows Center for Preventing Educational Risk (2017).

Intensifying Mathematics Tiers 2 and 3 Interventions for Students' MD and MLD

The primary focus of this book across the chapters is on how interventions can be planned for and intensified for Tier 2 and Tier 3 students. The challenge is to determine how we can provide quality instruction to help students with MD and MLD become more successful with mathematics learning. We provide a summary of important ways instruction can be intensified so it is responsive to struggling students. Additionally, you will read about many of these ways in subsequent chapters; here we provide an overview.

To begin, we ask the questions "Who needs intensive mathematics interventions?" and "What key ingredients are possible for intensifying interventions?" Students with persistent, chronic low mathematics performance in core or Tier 1 instruction are good candidates for intensive interventions. One key ingredient involves using cognitive strategies that support cognitive processing, such as strategies to support remembering steps and procedures for finding solutions to problems and solving basic facts. Self-regulation strategies, such as asking oneself to recall word-problem-solving procedures or using a checklist when steps are used, can be useful to regulate one's own learning in conjunction with cognitive strategies (Vaughn, Wanzek, Murray, & Roberts, 2012). Another key ingredient is using explicit, systematic instruction, which is described in this section. Yet another key ingredient is to control task difficulty starting with smaller "chunks" of content to reduce overload on students' memory. Now we turn our attention to examples of ways to intensify interventions for Tier 2 and Tier 3 for students who have MD and MLD. We discuss universal screening to identify struggling students, evidence-based practices for interventions, progress monitoring to determine student response to the intervention, vocabulary knowledge, mathematical misconceptions, and finally the ADAPT framework, which represents a process for identifying appropriate ways to adapt or change instruction to better address an individual student's needs.

Universal Screening for Mathematics Identification

Universal screening is an important feature of the RTI model because this process is used to identify students at risk for academic difficulties through the use of brief assessment measures (Jenkins, Hudson, & Johnson, 2007). Students whose scores on mathematics screeners fall below a designated "cut score" such as below the 25th percentile, that is, below average performance, are identified as in need of intensive intervention, which may be Tier 2 or Tier 3 depending on each student's individual performance and mathematics problems.

Evidence-Based Practices

Evidence-based practices (EBPs) refer to the continuous use of effective instructional routines, which have been shown through carefully constructed research studies to improve the mathematics performance of students with MD and MLD. A great deal of research has been conducted for many years on EBPs, and now we see these EBPs incorporated into Tier 2 and Tier 3 mathematics interventions. For example, Gersten and his colleagues (2009) conducted reviews of literature to determine those EBPs that held the most promise for improving the mathematics performance of students with MD and MLD. The EBPs they identified included (1) explicit and systematic instruction, consisting of the teacher's verbalizations (thinking aloud) of his or her thinking process for solving problems for students to hear, guided practice, corrective feedback, and frequent cumulative review; (2) word-problem-solving instruction based on common underlying structures or types of schema; (3) practice opportunities with the use of representations of mathematical ideas; and (4) 10 minutes of daily practice for students to develop fluent retrieval of basic facts (number combinations).

In another classic review of the literature on practices for students with MLD, Swanson, Hoskyn, and Lee (1999) found that, in addition to explicit, systematic instruction, cognitive strategy instruction (CSI) is effective for teaching, for example, basic facts and word-problem solving. CSI consists of cognitive strategies (steps or a routine for solving problems) and metacognitive strategies (self-questioning) for the learner to check his or her understanding of the instructional steps or routine (Montague & Dietz, 2009).

Finally, Swanson and his colleagues (1999) found the following instructional practices to be effective, and thus they are included in many interventions for students with MD and MLD: (1) breaking down tasks and providing step-by-step prompts, (2) asking process or content questions of students, (3) sequencing tasks from easy to difficult and teaching prerequisite skills, (4) delivering instruction via multimedia, (5) delivering instruction to a small group, and (6) reminding students to use strategies. We know a great deal about what constitutes EBPs; intensive interventions include many of these practices, as you will read about in the remaining chapters.

Progress Monitoring

Progress monitoring (PM) is also an essential feature of RTI, as it measures students' mathematics responsiveness to interventions. PM measures can assess students' understanding of lessons' objectives and content and can show students' mathematical misconceptions. There are three key features of PM to determine student responsiveness to mathematics interventions. First, specific skills that are being taught are measured. Second, the procedures are systematic, meaning that administration of PM measures is done the same way each time they are given. Third, PM measures are given consistently and frequently. For instance, Tier 2 students might receive a PM measure weekly, whereas Tier 3 students might receive PM twice a week. These PM measures can be implemented in a small group. Particular attention should be given to selecting evidence-based PM measures and to administering the measures as intended (e.g., following administration and scoring guidelines).

Vocabulary

Vocabulary knowledge is central to making sense of mathematics (NGA & CCSSO, 2010). The lack of vocabulary knowledge can negatively affect students' learning of new content (Fisher & Frey, 2008; Powell & Nelson, 2017). Because mathematics terms are unlikely to be used during daily conversation, they are challenging to learn. Therefore, explicitly teaching vocabulary terms is highly recommended so that when students encounter the terms they can understand their meaning and how they are associated with the mathematics (Dunston & Tyminski, 2013). Developing mathematical vocabulary knowledge allows struggling learners to expand their abstract algebraic reasoning and move beyond mathematical operations to solve word problems.

In the *Principles and Standards for School Mathematics* (NCTM, 2000), it was noted that the ability to communicate mathematically should be addressed in all areas of assessment and instruction. Clearly vocabulary, or the knowledge of words and their meanings, is a critical component of mathematics communication (Monroe, 2006). Many years ago, yet still relevant for today, Wiig and Semel (1984) commented that mathematics is "conceptually dense," meaning that students must comprehend the meaning of terms and mathematical symbols because, unlike in reading, there are few context clues to help aid in meaning. Other researchers agree (Miller, 1993; Schell, 1982), noting that mathematics language is complex and particularly abstract.

Several authorities (Miller, 1993; Monroe & Orme, 2002) have noted that unfamiliar vocabulary is a leading cause of mathematics difficulties. Bryant, Bryant, and Hammill (2000) identified difficulties with the language of mathematics as a distinguishing characteristic of MLD. Capps and Cox (1991) suggested that the language of mathematics must be directly taught during the course of a mathematics lesson. Monroe (1998) agreed, noting that mathematics vocabulary cannot be taught incidentally.

Misconceptions

Mathematical *misconceptions* are faulty and incorrect ideas resulting from students' misunderstanding about a mathematical idea or concept. In some cases, the term *errors* is used as a way to suggest that student thinking is faulty, but not all errors are misconceptions; rather, errors could be attributed to careless work, a solution that is lacking a step, or incorrect recall of the solution to a basic fact (e.g., 4 + 9 = 13 and not 12).

Misconceptions are usually based on faulty thinking about generalizations or rules or misunderstanding about the structure of the mathematics. For example, when multiplying two numbers (e.g., 40 • 6 = ?) where the multiplicand has a 0 in the unit place, sometimes students are told to multiply the 2 nonzero numbers, in this case 4 and 6, and then to "bring down the zero." Although this procedure might yield the correct solution, the procedure does not help students learn about place value for multiplying numbers with a 0 in the ones place. Despite teachers' best intentions, the "tricks" or "shortcuts" they teach contribute to students' misconceptions.

For students with MD and MLD, interventions must be mathematically correct with both teachers and students using mathematically precise language. For example, a rhombus is not a diamond and the alligator's opened mouth pointing in a specific direction does not convey the mathematically accurate use of the "greater than" or "less than" signs. Moreover, incorrect use of representations, such as manipulatives, does not automatically mean students conceptually grasp the problems for which the manipulatives are being used. Rather, students should be able to describe their use of the manipulatives to represent a problem or concept.

As another example, misconceptions about the meaning of the equal sign as an operator symbol rather than as a relational symbol leads to errors. For instance, when given nontraditional equations such as 6 = ____ + 2, students might think that the answer is "8" because 6 + 2 = 8.

These are but a few of the many mathematical misconceptions that students who struggle with mathematics exhibit. Teachers must understand what misconceptions exist for the content they teach and avoid tricks or shortcuts that could be causing misconceptions to fester. In fact, students often have to be explicitly taught ways to think about the mathematics that reduce or prevent misconceptions (Bamberger & Oberdorf, 2010).

ADAPT Framework

Adaptations help students with MD and MLD participate in classroom discussions and learn mathematical skills and concepts, which are emphasized in the CCSSM (NGA & CCSSO, 2010). We define adaptations as any alterations that are made to a lesson or scaffolds that are added to a lesson to account for (1) a lack of prerequisite skills necessary to learn new concepts and skills being taught or (2) struggles that students encounter during a lesson. For struggling students, teachers and mathematics interventionists must be aware of their students' mathematical strengths and struggles, so that they can plan ways to provide much-needed support. This support can occur in the form of making adaptations to instructional delivery, instructional activities, instructional content, and instructional materials.

By way of illustration, at the beginning of any new work we do in mathematics research, we usually conduct at least one focus group meeting with teachers and/or conduct teacher interviews. There are many purposes of such meetings and interviews, but perhaps the most important information we receive has to do with teachers' perceptions of the students they work with, specifically, how their students struggle with the grade-level content being taught. What we hear most often is that there are usually several students in their classes (more if there are students with MLD in the class) who do not have the skills that the teacher feels are needed to learn the content that is being taught; that is, they are ill prepared to meet the demands of the assignments.

The skills at the primary grade are foundational, such as having basic number sense, including numeral recognition, knowledge of magnitude, number sequences, place value, and so forth. At the upper elementary level, some students may still struggle with components of number sense, but they may also have difficulty with number combinations or anything having to do with rational numbers. At the middle and high school levels, earlier concepts and skills such as fractions, and ratios and proportions, may be lacking, which can impede the ability to be successful with more advanced mathematics. Teachers share with us that the students in their classes who do not have the basic mathematics prerequisites have little chance of being successful mastering the new skills and concepts being taught. Thus, the ADAPT framework can be a useful tool to promote understanding and knowledge.

The steps of the ADAPT framework are easy to follow. First, teachers Ask, "What am I requiring the student to do?" For example, the teacher thinks, "The students have to solve five computation problems written on the board that involve adding two 2-digit numbers with regrouping to the tens place." Second, the teacher Determines the prerequisite skills of the task. In our example, students typically have to listen to the teacher's explanations (i.e., attend to what is being said), see the items on the board, follow the teacher's instructions concerning what to do, have the fine motor skills needed to write the problems on a sheet of paper with proper alignment, know number combinations, add single digits for each place value (ones, tens), regroup from the ones to the tens place, and retrace (check their work). Certainly, as you read this, you may be able to add a few prerequisites of your own.

Next, teachers Analyze the student's strengths and struggles, specifically as they relate to the noted prerequisite skills. Which of the skills can the student accomplish readily (strengths), and which will impede their ability to do the task (struggles)? If any of the prerequisite skills are designated as struggles, the lesson will pose problems for the student and some changes (i.e., instructional adaptations) need to be made. To continue with our example, let's say that the student has difficulties or struggles with the concept of regrouping ten ones to a group of ten so that the two 2-digit numbers can be added correctly. But the student knows his number combinations, which is a strength.

Adaptations take place in the fourth step, which is Propose and implement adaptations from among the four adaptations categories. Those categories are Instructional Content (WHAT is being taught, the skills and concepts that are the focus of teaching and learning), Instructional Delivery (HOW the lesson is being taught, that is, the procedures and routines used to teach the lesson), Instructional Materials (TOOLS used during teaching and learning, such as supplemental aids used to teach and reinforce skills and concepts), and Instructional Activity (OTHER lessons that can be used to meet the same objective). With our example, the teacher decides to work with the student on regrouping using base-ten models or base-ten blocks, which is an example of an instructional material to show addition of the two 2-digit numbers beginning in

TABLE 1.4. ADAPT Framework: Ways to Intensify Mathematics Interventions

Tier 2	➡	Tier 3
Instruction—Modeling/think-aloud: Conducted as part of instructional lessons initially (ID)	➡	**Instruction**—Modeling/think-aloud: Conducted as part of instructional lessons *throughout a unit of lessons* (ID)
Instruction—Practice: Opportunities to say, show, and write solutions; practice built into lessons (ID)	➡	**Instruction**—Practice: Increased opportunities to say, show, and write solutions; more practice built into lessons; game formats to increase practice opportunities and increase motivation (ID)
Grouping: Five to six students with one teacher (ID)	➡	**Grouping:** One to three students with one teacher (ID)
Dosage: 3 days per week, 25- to 30-minute sessions (ID)	➡	**Dosage:** 5 days per week, 45- to 60-minute sessions (ID)
Instructional content: Task analyzed (IC)	➡	**Instructional content:** More task analyzed; smaller instructional steps for teaching whole-number computation; facts taught in smaller groups—just +0, just +1, +0 and +1 together (IC)
Vocabulary: Math terms reviewed within context of lesson (IC)	➡	**Vocabulary:** Math terms explicitly taught using vocabulary strategies (e.g., word mapping); connections between student language and math language: "plus" means "add" (IC)
Representations: Concrete–pictorial–abstract; move away from concrete and emphasize more pictorial/abstract/symbolic (IM)	➡	**Representations:** Concrete–pictorial–abstract/symbolic; remain with the three levels; gradually fade concrete (IM)
Progress monitoring: Weekly	➡	**Progress monitoring:** Two times per week

Note. ID, instructional delivery; IC, instructional content; IM, instructional material.

the ones place and regrouping a group of ten to the tens place. The teacher models the calculation process using the base-ten models, and the student practices along with the teacher. The problem to be worked is 36 + 48 = ? The following steps show how the teacher proceeds with modeling the calculation using base-ten models: 6 + 8 = 14, regroup one group of ten ones to the tens place, which leaves four ones in the ones place, 3 tens + 4 tens + 1 ten = 8 tens, 36 + 48 = 84.

Finally, after adaptations have been implemented, the teacher administers a Test to determine if the adaptations helped the student to accomplish the task. To finish our example, the teacher decides to give the student five problems, two digits + two digits with regrouping, to see if the student can now do the calculations correctly. In Table 1.4, we provide examples of how interventions can be intensified using the ADAPT framework in terms of instructional delivery, instructional content, and instructional materials. An example of adapting progress monitoring frequency is also provided. Examples of how to implement the ADAPT framework are presented in Chapters 5, 9, and 10. For the remaining chapters, think about how you can apply the ADAPT framework to some of the activities.

To summarize, some students in classrooms bring challenges that may be disability related or content specific. As teachers, we often incorrectly assume that all of the students in our classroom have the prerequisite skills needed to address the daily tasks we assign to them. However, we know through teacher focus-group meetings and teacher interviews that this is not always the case; one or two students (or more) face challenges that interfere with their ability to perform a lesson's task as prepared and delivered.

Conclusion

In conclusion, across the elementary and secondary grade levels, universal screening procedures, ongoing interventions utilizing EBPs, vocabulary instruction, and PM measures can be used to help struggling students improve their mathematics performance in Tier 2 and Tier 3 and reduce or eliminate their misconceptions about concepts and procedures. A tiered system can provide differentiated support, such as in Tier 1, and intensified interventions for Tier 2 and Tier 3, to students identified as having MD or MLD. As you read the remaining chapters in this book, consider how authors of the other chapters apply the content described in this chapter to their mathematics topic. We hope you enjoy the book!

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