

Preface

This book was written to provide a better understanding of the structural equation modeling (SEM) approach and how it can be used to conduct meta-analysis. You are encouraged to take a course in SEM to become familiar with SEM software and computer output. The terminology used in SEM can often be a challenge, so its use has been minimized here and meanings explained where necessary.

The first chapter provides a historical overview of the synthesis of research findings. Research synthesis dates back to the 1800s and has a rich history in the medical field; thus, medical journals have extensive coverage of the methods used. The terminology today has changed in the social sciences, where synthesis of research is called meta-analysis. The concept of research bias is also introduced in this chapter.

Chapter 2 covers the types of research literature searches, namely, integrative review, scoping review, semi-systematic review, and systematic review. Systematic review has gained the most popularity, especially among university library staff. An attempt has also been made in this book to list some of the literature search software. References are provided for ease of access.

Chapter 3 covers the many different meta-analysis software packages that have been developed over the years. Some of these are commercial packages while others (R library) are free. This chapter also includes a sampling of teaching and learning meta-analysis software. The discussion of effect size calculator software provides much-needed information for obtaining effect sizes in meta-analysis.

Chapter 4 continues the meta-analysis journey by describing the background and use of meta-analysis statistics. Examples using the `compute.es()` and `tes()` functions are presented. In addition, two R program examples highlight how the statistics are computed. The chapter appendix contains the programs.

Chapter 5 explains the basic SEM modeling steps: model specification, model identification, model estimation, model testing, and model modification. The basic

steps are highlighted in other SEM books. Of specific interest is how to conduct path analysis, confirmatory factor analysis, and SEM. The LISREL-SIMPLIS, Mplus, and R packages are used to illustrate the different analyses.

Chapter 6 provides the assumptions, steps, and different model types used in meta-analysis. The different model types are univariate, multivariate, meta-regression, and meta-analysis SEM (MASEM). Each type is described in detail with an example. A discussion of fixed and random effects rounds out the chapter.

Chapter 7 gives an in-depth presentation of a path model MASEM using the LISREL-SIMPLIS program. The explanation of the data, program, and computer output is provided for a better understanding of the meta-analysis application. The chapter appendix includes Mplus and R lavaan programs.

Chapter 8 gives an in-depth presentation of a confirmatory factor model MASEM using the LISREL-SIMPLIS program. The explanation of the data, program, and computer output is provided for a better understanding of the meta-analysis application. The chapter appendix includes LISREL-SIMPLIS, Mplus, and R programs.

Chapter 9 presents the MASEM approach. This is the core topic of this book, which can now be presented following introduction of the concepts and programs in earlier chapters. The LISREL-SIMPLIS program highlights how this can be accomplished. Several LISREL-SIMPLIS programs are included in the chapter appendix. Cooper (2010) provided helpful guidelines for reporting results from a meta-analysis study based on current APA standards. In later chapters of the book, other programs, such as metaSEM, are presented.

Chapter 10 gives insight into why meta-analysis should be considered a multilevel method. The reason is that study data are collected over several years; hence time is a nested effect. Multilevel regression is explained, along with the reticular action model (RAM). RAM methods were used in 2015 to conduct MASEM with the metaSEM package in R. The chapter therefore includes metaSEM examples. The lme4 package in R provides an example of multilevel linear mixed methods. Finally, a multilevel confirmatory factor analysis (CFA) model is presented in a LISREL-PRELIS application.

Chapter 11 discusses the many SEM and MASEM issues. These include the literature search process, publication bias, heterogeneity of effect sizes across studies, sample size differences between studies, correlation standard error issues, two-stage approach, conversion of covariance to correlation or vice versa, and power. Many of these topics have been discussed in detail in books and in a long list of articles in medical and social science journals. The chapter appendix contains the programs.

Chapter 12 summarizes the MASEM method. We are reminded of the steps a researcher takes to conduct meta-analysis. The steps are arduous and time consuming. Some books and articles have even suggested a 1- to 2-year timetable to conduct a meta-analytic study: Step 1, define topics and select studies; Step 2, determine statistical and effect size; Step 3, create a meta-analysis data set; Step 4, select a software

package; Step 5, run MASEM and report results. Many researchers have also suggested the importance of outlining these steps so that future researchers can update the meta-analysis results. Key concepts from Chapter 11 are revisited in Chapter 12, along with the important standards in PRISMA. This chapter also covers the funnel plot, causes of publication bias, heterogeneity issues, different sample sizes, missing data, correlation standard errors, and power. A chapter appendix provides additional information.

The various computer programs are placed in boxes within the chapters to more clearly delineate the program code. The computer output is indicated in a Courier New font that permits proper alignment of results. Many more programs are provided in an appendix at the end of each chapter. All programs are saved in Notepad files so they can easily be opened/copied/pasted into corresponding LISREL, Mplus, or R software.

The different computer programs and data sets used in the book are available on the book's website at www.guilford.com/schumacker-materials. Several chapters of this book contain computer programs and data sets using LISREL, Mplus, or R, and in a few cases SPSS and SAS.

The LISREL commercial licensed program is available at <https://ssilive.com/license/lisrel>. If you are taking a class or teaching a class using LISREL, special educator benefits are available.

The Mplus commercial licensed program is available at www.statmodel.com. An Mplus Version 9 demo is available at no cost for the Windows, Mac OS X, and Linux operating systems. The demo version is limited by the number of variables that can be used in an analysis:

- number of dependent variables: 6
- number of independent variables: 2
- number of between (moderator) variables in two-level analysis: 2
- number of continuous latent variables in time series analysis: 2

The R program is a free open source software for Linux, Mac OS X, or Windows and is available at <https://cran.r-project.org>.

Chapter 6

Meta-Analysis Assumptions, Steps, and Model Types

This chapter covers the assumptions that are generally made by a researcher when conducting meta-analysis: causal effect, inclusion criteria, missing data, publication bias, extraneous variables, and other important study variable information. More importantly, the chapter provides information about pooling data from selected studies, and the basic data assumptions related to conducting SEM research (e.g., determinant of matrix, multivariate normality). The meta-analysis steps and guidelines are also described, with corresponding references. The model types (i.e., univariate, multivariate, meta-regression, and MASEM) are covered to provide a background and an understanding of the different approaches a researcher could take in meta-analysis using SEM. MASEM specifically addresses how SEM pools correlation matrices across selected studies. Fixed and random effects are also briefly discussed.

ASSUMPTIONS

Meta-analysis in general involves some important assumptions. Additional assumptions are included when conducting meta-analysis using correlation matrices in structural equation modeling (MASEM). These can be summarized as causal assumptions, inclusion criteria, missing data, publication bias, and auxiliary variables (i.e., control, moderator, or instruments). Structural equation modeling (SEM) can contain variables in a correlation matrix, which can define one or more latent variables depending on the study and type of SEM model: path, factor, or structural (Schumacker & Whittaker, 2027). These different SEM models have been used in various studies testing theories related to specific topics. Both the basic meta-analysis assumptions and the additional MASEM assumptions are important when using structural equation modeling in meta-analysis.

The basic meta-analysis assumptions are (Allen, 2019; Steinmetz & Block, 2022):

1. The pooled data being used are from studies with similar demographic data.
2. The pooled data being used are from studies with similar analysis methods and results.
3. Study selection has sufficient sample size.
4. Criteria for selecting studies are specified to avoid leaving out pertinent studies, and to only include relevant studies.
5. The topic and related research questions are expressly stated, illustrating that the pooled data synthesis relates to the topic and research questions.
6. Pooled data analysis assists in reducing Type II errors (false negative; significant in reality but found nonsignificant) and identifying Type I errors (false positive; nonsignificant in reality but found significant).

These meta-analysis assumptions are keenly related to publication bias (e.g., journal impact factor, professional association journal, years of journal publication), study selection, reporting of statistical analysis in the studies, heterogeneity from outlier studies and/or sample size to effect size, small study selection, omission of any relevant study, and study selection that is on target with the topic and research questions. When pooling data the possibility exists that the combined data indicates a Type II error or possibility of a Type I error. Larger effect sizes and sample sizes alleviate most concerns related to Type II error in meta-analysis. Type I error is generally reduced by increasing the alpha level of significance from .05 to .01. When combining sample data, meta-analysis generally reduces the confidence interval around any statistical estimation, which increases the accuracy of the statistical parameter. The best practice in meta-analysis is to identify pertinent studies with effect sizes.

The additional MASEM concerns include:

1. Does the meta-analysis meet assumptions for using a correlation and/or variance-covariance pooled matrix?
 - a. Positive determinant/eigenvalues
 - b. No outliers
 - c. No missing data
 - d. Multivariate normality
 - e. If using a Pearson correlation matrix, then assume continuous variables; linear, normal distributions; and independent observations.
2. Does the synthesis of pooled data relate to a theoretical model and/or constructs?

3. Does the pooled data analysis consider contribution of moderator variables (i.e., relationships between and among variables)?
4. Are the theoretical models being tested in the selected studies relatable?

The MASEM assumptions are related to the general use of SEM as outlined in Schumacker and Whittaker (2027) as well as the analysis of the pooled correlation matrix. Moreover, the presence of descriptive statistics (e.g., sample size, mean, standard deviation), correlation matrix, and a graph of the theoretical model greatly enhances the ability to pool study data correlations for meta-analysis.

META-ANALYSIS STEPS

The modeling approach in MASEM has different steps compared to other meta-analysis methods. Based on Viswesvaran and Ones (1995) and Cheung (2015a) guidelines, the steps recommended for conducting MASEM are:

- a. Review the key articles relevant to the topic.
- b. Formulate problem statements as a guide to determine constructs and relations at the early stages.
- c. Identify all studies that belong to the categories of interest in the topic.
- d. Identify measures for each construct among the relations of interest.
- e. Formulate the research questions from the studies.
- f. Gather statistical data and coefficients from the studies.
- g. Estimate true score correlation and pool correlation coefficients or the composite scores for the variables and their items or the constructs.
- h. Use appropriate techniques to analyze the hypothesized measurement model or the structural model.
- i. Interpret results, draw conclusions, and summarize.

O'Leary (2021, pp. 524–525), proposed six stages for performing meta-analysis:

1. Problem statements and research question formulation.
2. Inclusion/exclusion criteria should be well discussed and determined.
3. Search strategy should be explicitly stated and strictly followed.
4. Data and findings of the selected studies should be critically evaluated.
5. Statistical analysis can be performed on the gathered information.
6. Results should be synthesized and interpreted.

Hansen et al. (2022) followed the previous authors' suggestions with their own eight steps:

1. Define the research question.
2. Literature search: systematic, reproducible search strategy.
 - a. Search strategy should be explicitly stated and strictly followed.
 - b. Inclusion criteria stated and sample composition.
 - c. Specify any "gray literature" from less known articles.
3. Choice of effect size measures (d , z , transformed r to z).
4. Choice of analytic method (e.g., univariate, meta-regression, MASEM, qualitative).
5. Choice of software (e.g., Stata, SPSS, SAS, R, Excel, RevMan, Comprehensive MA).
6. Coding of effect sizes (e.g., code sheet with study name, effect size, sample size).
 - a. Inclusion of moderator (mixed effects meta-analysis) or control variables.
 - b. Treatment of multiple effect sizes in study.
7. Statistical analysis of data (outliers, publication bias, model choice).
8. Reporting results (mean effect sizes, standard errors, confidence intervals).
 - a. Use of a checklist.
 - b. Include study sample sizes, heterogeneity measures, method used.
 - c. Interpretation related to future research direction.
 - d. Open-science practice with open source repository for data sets and coding.

All of these references have much in common regarding the steps a researcher would take, the article by Hansen et al. (2022) being the most current and complete in providing additional citations for the steps discussed. The main objective of the Stage 1 analysis is to pool the correlation (or covariance) matrices together. Stage 2 then uses the correlation (covariance) matrix in the model analysis. Most applications of MASEM in Stage 2 use the pooled correlation matrix as if it were an observed correlation matrix to fit structural equation models. Discussed next are the different model types used in MASEM.

MODEL TYPES

The meta-analysis models are usually characterized as univariate, multivariate, meta-regression, MASEM, qualitative, or Bayesian (Schmid et al., 2020). Schmid

et al.'s book covers the full range of statistical methods in meta-analysis. It includes thousands of references, data, code, and examples to enhance a reader's understanding of the depth and breadth of the models. Our discussion in this book focuses on MASEM and does not cover the qualitative (Levitt, 2018; Timulak, 2009) or Bayesian methods (Garrett et al., 2024; Reis et al., 2023).

Univariate

The *univariate* method of meta-analysis pools the data from n independent studies, applies a fixed or random effects model, and examines the within-study variance. Most of the independent studies would have been from different researchers and included different methods, and are therefore generally assumed to be heterogeneous. The random effects model in this case would produce a more conservative result and would not include the between-study effects.

Consider an example univariate meta-analysis data set from `install.packages("metadat")` in R (R Team, 2024) that examines the correlation between attendance and grades from the data set `dat.crede2010` (Credé et al., 2010):

```
>install.packages("metadat")
>library(metafor)
>dat = dat.crede2010
>head(dat, 10)
```

	studyid	year	source	sampleid	criterion	class	ni	ri
1	1	2009	dissertation	1	grade	nonscience	76	0.8860
2	2	1975	journal	1	grade	nonscience	297	0.3000
3	3	2007	dissertation	1	gpa	<NA>	191	0.7200
4	4	1989	journal	1	grade	nonscience	265	0.4750
5	4	1989	journal	2	grade	nonscience	154	0.3340
6	5	2008	journal	1	grade	nonscience	162	0.6150
7	6	1999	journal	1	grade	nonscience	28	0.1450
8	6	1999	journal	2	grade	nonscience	33	0.2300
9	6	1999	journal	3	grade	nonscience	47	0.2700
10	6	1999	journal	4	grade	nonscience	25	-0.0228

The `studyid` is the study number (1 to 6); `year` is the publication year; `source` is as listed; `sampleid` is sample within study; `criterion` is grade or gpa; `class` is science/nonscience; `ni` is the sample size; and `ri` is the observed correlation. There were 68 studies included in this data set about the relationship between class attendance of college students and their performance (i.e., grade) in the class or their overall grade point average (gpa). Some studies included multiple samples and hence the data set actually contains 97 correlation coefficients. The meta-analysis is on the correlation coefficients (`ri`).

```

## calculate r-to-z transformation sampling variances

>dat = escalc(measure="ZCOR", ri = ri, ni = ni, data=dat)

## meta-analysis for correlation of attendance and grades

>res = rma(yi, vi, data=dat, subset=criterion=="grade")
>res

Random-Effects Model (k = 67; tau^2 estimator: REML)

tau^2 (amount of total heterogeneity): 0.0511 (SE = 0.0104)
tau (square root of estimated tau^2 value):      0.2261
I^2 (total heterogeneity / total variability): 93.83%
H^2 (total variability / sampling variability): 16.21

Test for Heterogeneity:
Q(df = 66) = 1068.7213, p-val < .0001

Model Results:

estimate   se    zval    pval   ci.lb  ci.ub
  0.4547 0.0300 15.1343 <.0001  .3958 0.5136 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## estimated average correlation with 95%CI (lb; ub)/PI(lb; ub)

>predict(res, transf = transf.ztor, digits=2)

pred ci.lb ci.ub pi.lb pi.ub
  0.43 0.38 0.47 0.01 0.72

```

The meta-analytic estimated correlation was 0.4547 from the 97 study correlations. The predicted average correlation was 0.43. Both the estimated and predicted average correlations fell in the 95% confidence intervals (0.3958, 0.5136 and 0.38, 0.47, respectively). A researcher could include in the meta-analysis that there is a strong relationship between attendance and grades.

Multivariate

A multivariate method was introduced to analyze the n -independent study effect sizes. When the within-study variance-covariance matrices for S_i are known, then the random effects model for y_i and μ_i can be specified as

$$y_i \sim N(\mu_i, S_i)$$

$$\mu_i \sim N(\mu, \mathbf{T})$$

where μ_i is the underlying true effect sizes, $\mu = (\mu_1, \dots, \mu_p)\mathbf{T}$, which contains the overall effect sizes, and $\mathbf{T}$ is the between-study variance-covariance matrix. This may limit study selection for effect sizes if the within- and between-study correlations are not included.

Consider an example multivariate meta-analytic data set from library(metadata) called dat.craft2003 that contained 10 studies examining competitive state anxiety and sport performance (Craft et al., 2003):

```
>library(metadata)
>dat = dat.craft2003
>head(dat, 10)
```

	study	ni	sport	ri	var1	var2
1	1	142	I	-0.55	acog	perf
2	1	142	I	-0.48	asom	perf
3	1	142	I	0.66	conf	perf
4	1	142	I	0.47	acog	asom
5	1	142	I	-0.38	acog	conf
6	1	142	I	-0.46	asom	conf
7	3	37	I	0.53	acog	perf
8	3	37	I	-0.12	asom	perf
9	3	37	I	0.03	conf	perf
10	3	37	I	0.52	acog	asom

The study variable is a study number ranging from 1 to 3; ni is sample size; sport is either T (team) or I (individual); ri = correlation coefficient; var1 is variable acog (cognitive anxiety), asom (somatic anxiety), or conf (self-confidence); var2 is variable perf (performance), asom (somatic anxiety), or conf (self-confidence). The R code is:

```
## construct dataset and var-cov matrix of the correlations

>tmp = rcalc(ri ~ var1 + var2 | study, ni=ni, data=dat)
>V = tmp$V
>dat = tmp$dat

## multivariate random-effects model

>res = rma.mv(yi, V, mods = ~ var1.var2 - 1, random = ~
+var1.var2 | study, struct="UN", data=dat)
```

The output is:

```
>res
```

```
Multivariate Meta-Analysis Model (k = 51; method: REML)
```

```
Variance Components:
```

```
outer factor: study (nlvls = 9)
```

```
inner factor: var1.var2 (nlvls = 6)
```

	estim	sqrt	k.lvl	fixed	level
tau^2.1	0.0047	0.0683	9	no	acog.asom
tau^2.2	0.0125	0.1119	8	no	acog.conf
tau^2.3	0.1611	0.4014	9	no	acog.perf
tau^2.4	0.0111	0.1052	8	no	asom.conf
tau^2.5	0.0604	0.2459	9	no	asom.perf
tau^2.6	0.0468	0.2163	8	no	conf.perf

	rho.acg.s	rho.acg.c	rho.acg.p	rho.asm.c	rho.asm.p
rho.cnf.					
acog.asom	1				
acog.conf	-0.6961	1			
acog.perf	0.5491	0.0432	1		
asom.conf	-0.0891	0.4193	0.3532	1	
asom.perf	0.4604	-0.0495	0.9497	0.2688	1
conf.perf	-0.9345	0.7023	-0.6178	-0.1311	-0.5969

```
Test for Residual Heterogeneity:
```

```
QE(df = 45) = 334.8358, p-val < .0001
```

```
Test of Moderators (coefficients 1:6):
```

```
QM(df = 6) = 596.7727, p-val < .0001
```

```
Model Results:
```

	estimate	se	zval	pval	ci.lb	ci.ub
var1.var2acog.asom	0.5671	0.0367	15.4640	<.0001	0.4953	0.6390

var1.var2acog.conf	-0.4888	0.0509	-9.6048	<.0001	-0.5886	-0.3891

var1.var2acog.perf	-0.0600	0.1408	-0.4264	0.6698	-0.3359	0.2159
var1.var2asom.conf	-0.4750	0.0506	-9.3901	<.0001	-0.5741	-0.3758

var1.var2asom.perf	-0.1423	0.0917	-1.5527	0.1205	-0.3220	0.0373
var1.var2conf.perf	0.3167	0.0847	3.7393	0.0002	0.1507	0.4827

```
---
```

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The model results indicated that *acog* (cognitive anxiety) and *asom* (somatic anxiety) shared variance (.5671; $z = 15.4640$; $p < .0001$). Likewise, *acog* and *conf* (self-confidence) and *asom* and *conf* shared variance, respectively (–.4888; $z = -9.6048$; $p < .0001$ and –.4750; $z = -9.3901$; $p < .0001$). Self-confidence (*conf*) was also related to *perf* (sport performance): (.3167; $z = 3.7393$; $p < .0002$). Self-confidence was negatively related to cognitive and somatic anxiety. Self-confidence was positively related to sport performance. We can therefore conclude that a synthesis of the 10 studies indicated that anxiety was significant in moderating the relationship between self-confidence and sport performance.

Meta-regression

The meta-regression method parallels the basic multiple regression technique except that the outcome, covariates, and any moderator variables are measured at the study unit of analysis. Since studies differ in sample size, a sample weighting scheme is often used in either the fixed effect or the random effect meta-analysis. A common application is to assess the association between study effect size (Y) and sample size (covariate X), that is, the impact of the sample size on the effect size in the study. A line plot of the effect size compared to the sample size with a confidence interval around the regression line is useful in visualizing the relationship. The closer the study value is to the regression prediction line, the smaller the difference between the observed and predicted Y (effect size) estimates.

An example regression meta-analytic data set from library(*metadat*) called *dat.aloe2013* researched studies that used regression models to examine the association between some measure of perceived supervision quality and some work-related outcome (e.g., job satisfaction). It includes results from five studies (Mor Barak et al., 2009) with the data frame listed as:

```
>library(metadat)
> dat.aloe2013
```

		study	<i>n</i>	<i>tval</i>	preds	R^2
1	Abu-Bader	(2000)	218	4.61	4	0.240
2	Cole et al.	(2004)	232	6.19	7	0.455
3	Wallach & Mueller	(2006)	156	4.07	6	0.500
4	Weaver et al.	(2007)	382	–0.77	19	0.327
5	Acker	(2004)	259	1.16	15	0.117

The study variable is the author and year; n is the sample size; *tval* is the t -statistic for association between outcome and predictor; *preds* is the number of predictors in the regression model; and R^2 is the R -squared value in the regression model.

The regression meta-analysis for random effects is run in the R package *metafor*. The previous data set will again be used to calculate the partial correlation

coefficients (y_i) and the sample variance (v_i). The metafor package and data are listed as:

```
# compute partial correlation coefficients and sampling
  variances
>library(metafor)
>dat = dat.aloe2013
>dat = escalc(measure="PCOR", ti=tval, ni=n, mi=preds,
  data=dat)
>dat
```

	study	n	tval	preds	R2	yi	vi
1	Abu-Bader (2000)	218	4.61	4	0.240	0.3012	0.0039
2	Cole et al. (2004)	232	6.19	7	0.455	0.3822	0.0032
3	Wallach & Mueller (2006)	156	4.07	6	0.500	0.3163	0.0054
4	Weaver et al. (2007)	382	-0.77	19	0.327	-0.0404	0.0027
5	Acker (2004)	259	1.16	15	0.117	0.0742	0.0041

```
## random-effects model
>res = rma(yi, vi, data=dat)
>res
Random-Effects Model (k = 5; tau^2 estimator: REML)

tau^2 (estimated amount of total heterogeneity):
0.0298 (SE = 0.0237)

tau (square root of estimated tau^2 value):      0.1725
I^2 (total heterogeneity / total variability): 88.91%
H^2 (total variability / sampling variability): 9.01

Test for Heterogeneity:
Q(df = 4) = 40.1986, p-val < .0001

Model Results:

estimate   se   zval    pval  ci.lb  ci.ub
  0.2048 0.0820 2.4993 0.0124 0.0442 0.3655 *

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The regression meta-analysis yielded an average R^2 value = 0.2048 with significant heterogeneity ($Q(df = 4) = 40.1986, p\text{-val} < .0001$).

MASEM

The structural equation modeling method called MASEM generally involves the pooling of correlation matrices (Cheung, 2015a, 2015b). It is understood that the

variance-covariance matrices from studies can be pooled (averaged) and then converted to a correlation matrix for MASEM analysis (Jak, 2015). Since correlation and/or covariance matrices in select studies form the basis of MASEM, it is important to be able to convert them accordingly (Balduzzi et al., 2019; Harrer et al., 2021).

In R, the `cov2cor()` function converts a covariance matrix into a corresponding correlation matrix. Therefore, a researcher can add up all the terms in the variance-covariance matrices of the selected studies, then convert to a correlation matrix. This is preferred over adding the correlation coefficients from the studies and dividing by the number of studies.

Some selected studies may only report the descriptive statistics and a correlation matrix, so conversion to a variance-covariance matrix is necessary before pooling the data. To convert a $p \times p$ correlation matrix to a covariance matrix, you need the variances (or standard deviations) of the p variables. So, the `cor2cov()` function includes the correlation matrix and variable standard deviations, and indicated by

```
>cor2cov(cor.mat, sd)
```

where `cor.mat` is the correlation matrix and `sd` is a vector with the variable standard deviations.

This function can be understood by noting the **R** correlation matrix and **S** vector of standard deviations in the expressed function. The conversion is done as follows:

```
>cor2cov = function(R,S) { diag(S) %*% R %*% diag(S) }
```

where the `%*%` in the function represents multiplication of the matrices.

These two functions (`cov2cor`; `cor2cov`) are very helpful when working with correlation matrices or variance-covariance matrices in selected studies. Also, if multiple effects are reported in a study, adding up the variance-covariance terms and then converting to a correlation matrix would be helpful. Likewise, keeping the study variance-covariance matrix, then adding to the other study variance-covariance matrices before conversion to a correlation matrix is helpful. The R program example in the following box will help to clarify how this is done.

R Program

```
# convert correlation matrix to covariance matrix
# Correlation matrix
R = matrix(c(1.00,0.25,0.90,0.25,1.00,0.50,0.90,0.50,1.00),
           nrow=3,ncol=3)
R
# standard deviations
c = matrix(c(1,1,1,1,4,1,1,1,9),nrow=3,ncol=3)
```

```

c
# Covariance matrix
S = (c %*% R %*% c)
S

```

The MASEM analysis is simplified when the study variance-covariance matrices are available. Researchers understand that use of variance-covariance matrices permits addressing the scaling of variables (raw score scale), while correlation matrices are standardized so the variance is always 1.00 in the diagonal of the matrix.

In MASEM the issue when using a correlation matrix is that the model-implied matrix may not have exactly 1.00s in the diagonal of the matrix. Another issue in MASEM is that the number of variance and covariance terms in a correlation matrix is known to be $p(p + 1)/2$; however, with correlation matrices having a 1.00 in the diagonal, the variable variances do not provide any additional information. For example, given six (6) variables in a correlation matrix, $6(6 + 1)/2 = 6(7)/2 = 42/2 = 21$ elements in the matrix, but in meta-analysis practice the usable information is $6(6 - 1)/2 = 6(5)/2 = 30/2 = 15$ (the 6 variable variances are removed). This leads to incorrect standard errors and thus incorrect estimation of parameters when using a correlation matrix (Schumacker & Lomax, 2016).

The covariance matrix is generally expressed as

$$\Sigma(\theta) = \Lambda\Phi\Lambda' + \delta$$

where Λ is a matrix of factor loadings, δ is the matrix of measured variable errors, and Φ is the covariance of the latent variables. The log-likelihood ratio (LR) statistic tests the homogeneity of the covariance matrices, $H_0 : \Sigma_1 = \Sigma_2 = \Sigma_3 \dots = \Sigma_k$ from the selected studies. The asymptotic variance-covariance is generated and used as a weight matrix in those estimation methods, and in MASEM the pooled correlation matrix with the asymptotic covariance matrix is the preferred approach for analysis. Different estimation methods are available in SEM, with the weighted least squares and diagonally weighted least squares estimation methods preferred to address issues related to scaling and multivariate normality. Also, SEM provides goodness-of-fit statistics given different model fit criteria, as discussed in Schumacker & Lomax (2016; Chapter 5).

Problems arise in MASEM when pooling the correlation coefficients from selected studies. For example, the number and types of variables may be different, even for studies investigating the same theoretical model. This is due to researchers adding variables or modifying the originally published theoretical model with different types of variables. Sometimes the elements of a pooled correlation matrix are estimated with different sample sizes. Another problem arises when the correlation matrices from the studies do not contain all the same variables. A researcher might

be able to put together a complete correlation matrix that matches others from the selected studies. This can be accomplished by incorporating single variable results in some studies or averaging the correlations from other studies and inserting them in the missing correlation matrix. This approach would enable researchers to include the maximum number of studies for MASEM.

To pool correlation matrices in the R lavaan package, you would typically specify a model that uses the average correlation matrix across studies. When pooling matrices, you need to consider how to account for potential differences between studies (e.g., using a random effects model in MASEM).

An example of a MASEM data set used in the metaSEM package in R—the Becker94 correlation matrices—is shown below. The example uses six correlation matrices from studies where sample sizes were different:

```
>library(metaSEM);library(lavaan)
>data(Becker92)
>head(Becker92$data)
```

```
$'Berry (1957)'
```

	Math	Spatial	Verbal
Math	1.00	0.46	0.31
Spatial	0.46	1.00	0.19
Verbal	0.31	0.19	1.00

```
$'Rosenberg (1981)'
```

	Math	Spatial	Verbal
Math	1.00	0.46	0.55
Spatial	0.46	1.00	0.32
Verbal	0.55	0.32	1.00

```
$'Weiner 1 (1984)'
```

	Math	Spatial	Verbal
Math	1.000	0.397	0.402
Spatial	0.397	1.000	0.183
Verbal	0.402	0.183	1.000

```
$'Weiner 2 (1984)'
```

	Math	Spatial	Verbal
Math	1.000	0.266	0.567
Spatial	0.266	1.000	0.218
Verbal	0.567	0.218	1.000

```
$'Becker 1 (1978)'
```

	Math	Spatial	Verbal
Math	1.00	0.28	0.19
Spatial	0.28	1.00	0.18
Verbal	0.19	0.18	1.00

```

$'Becker 2 (1978)'
      Math Spatial Verbal
Math      1.00  0.47 -0.21
Spatial   0.47  1.00 -0.15
Verbal    -0.21 -0.15  1.00

```

Sample sizes were different for the six correlation matrices.

```

>head(Becker92$n)
[1] 103 69 69 70 153 74

```

The pooled correlation matrix was obtained using the `tssem1()` function rather than the previous R code to average the matrices. The sample sizes for each correlation matrix were combined and a fixed effects model specified (`method = "FEM"`).

```

# Pooled correlation matrix
>pool = tssem1(Becker94$data, Becker94$n, method="FEM")
>coef(pool)
      Math Spatial  Verbal
Math      1.0000000 0.3792568 0.2963633
Spatial   0.3792568 1.0000000 0.1645410
Verbal    0.2963633 0.1645410 1.0000000

```

One could then put this pooled correlation matrix in a LISREL-SIMPLIS program to obtain the factor loadings and fit indices for a CFA model with the average of the sample sizes ($538/6 = 89.7 \sim 90$). That program is in the box below.

LISREL-SIMPLIS Program

```

Becker 1992 Pooled Correlation Matrix
Observed variables Math Spatial Verbal
Sample Size 90
Correlation Matrix
1.0000000
0.3792568 1.0000000
0.2963633 0.1645410 1.0000000
Latent variable Ability
Math Spatial Verbal = Ability
Set variance of Ability = 1
Path Diagram
End of Problem

```

The confirmatory factor model is diagrammed in Figure 6.1.

A researcher would run the CFA for each of the study correlation matrices and the pooled correlation matrix. Finally, a table indicating the factor loadings from the

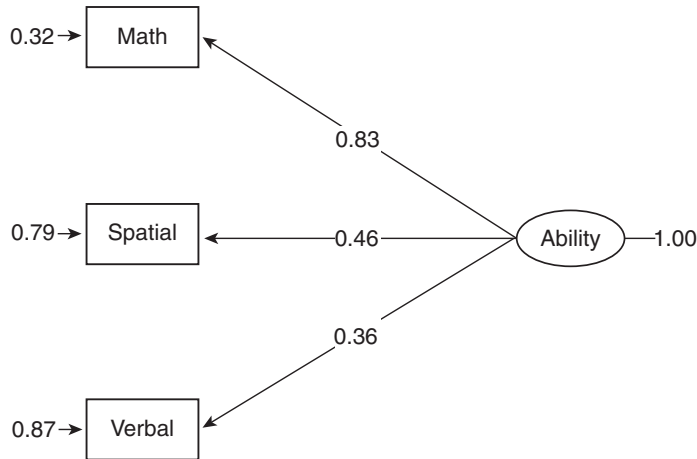


FIGURE 6.1. CFA diagram of the Becker94 pooled correlation matrix data.

individual studies along with the factor loadings from the pooled correlation analysis would be prepared.

FIXED VERSUS RANDOM EFFECTS IN META-ANALYSIS MODELS

The various books and articles on meta-analysis over the years have highlighted an important distinction between fixed and random effects that parallels a similar discussion in experimental research designs (Nikolakopoulou et al., 2014). Meta-analyses use either fixed or random effects models. A fixed effect meta-analysis assumes all studies are estimating the same (fixed) effect, whereas a random effects meta-analysis allows for differences in the effects from study to study. An experimental design, for example, might have three treatments: 0 mg caffeine, 15 mg caffeine, and 30 mg caffeine. In a fixed effects interpretation, the outcome would only be interpreted at each individual dosage. In a random effects interpretation, the outcome would vary between 0 and 30, with degrees of interpretation.

There have been statistics created to assess the impact of a fixed effect versus random effects approach to meta-analysis. For example, a meta-analysis with n independent studies would give μ_i as the underlying true effect size (such as transformed r to z) in study i ($i = 1, \dots, n$). Many published studies report estimates of the effect sizes and their within-study variances, which are defined as y_i and v_i (s^2). It is assumed that the y_i 's are approximately normally distributed with mean μ_i and variance σ^2 , respectively. Since the unknown σ^2 can be estimated by s^2 , these data are commonly modeled as $y_i \sim N(\mu_i, s^2)$, with s^2 treated as a known value. The assumption that the true μ_i 's are independently distributed as $\mu_i \sim N(\mu, \tau^2)$ is

specified, where μ is the true overall mean effect size across studies and τ^2 is the between-study variance. The collected n studies are defined to be homogeneous if their underlying true effect sizes are equal, that is, $\mu_i = \mu$ for all $i = 1, \dots, n$ independent studies, or equivalently $\tau^2 = 0$. If the studies are heterogeneous then the underlying true effect sizes vary, that is, $\tau^2 > 0$.

Besides the LR test in MASEM, another test of homogeneity of the independent studies is a Q statistic test, which follows a χ^2 distribution. H^2 statistics—the ratio of the standard deviation of the overall effect size from a random effects meta-analysis compared to the standard deviation from a fixed effect meta-analysis—are also proposed. Also, there is a reported I^2 value that provides the proportion of variance between studies attributed to heterogeneity rather than sampling error, with a value falling between 0 and 1. All three of these statistics are currently used in reporting meta-analysis results.

SUMMARY

Researchers using structural equation modeling generally test a theoretical model. SEM uses the correlation/partial correlation (variance-covariance) matrix to test the theoretical model that specifies the relations among the variables. The various relevant published studies would include a collection of different sample data applied to the theoretical model. Therefore, the pooling of the correlation matrices across a selection of these studies weighted by sample size provides a unique basis to conduct meta-analysis.

The MASEM approach yields an interpretation of overall study results in meta-analysis because it is based on a theoretical model. The level of accuracy is increased because the selected studies generally have large sample sizes, providing a better effect size to sample size ratio. MASEM using pooled correlation matrices may also benefit from the availability of effect sizes from the individual study results. MASEM would be using studies that have tested a theoretical model using correlation or variance-covariance matrices that are reported in the published studies.

The MASEM method will collect tests of parameter significance that might indicate small, medium, or large effects (partial eta-squared) given a complex theoretical model that might be different from results in a simpler designed study. The method also permits model testing for a variety of different theoretical models that have been postulated across numerous disciplines. This provides more avenues for meta-analysis on a given topic. It also provides the basis to confirm a theory based on the synthesis of the research literature.