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CHAPTER 4

Math Language

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In this chapter, we focus on *math language* and, more specifically, the *vocabulary* that is important to understand math. Math is its own language, but it is not a universal language (Barrow, 2014). In every language (e.g., English, Spanish, German, Mandarin), students must learn the academic language that allows them to understand and communicate (Schleppegrell, 2012). In this chapter, we focus on the academic language of math—the words, numerals, symbols, and other visuals—that describe the concepts and procedures of math.

MATH NUMERALS

In English (and most other languages), there are 10 digits that can be used to show all numerals. Those digits are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, with 0 likely the most important digit, as it allows for holding of place value. Have you ever tried multi-digit computation with Roman numerals (e.g., CCXVI + LIV)? A *numeral* is the symbol or name that represents a number. For example, 4, 105, and thirteen are all numerals. A *number* is how students envision numerals. For example, 4 crayons or 4 clips for "4." In this book, we use *number* to describe both numerals and numbers.

Students should understand numbers in three ways (McGregor, 2013): the numeral, the name, and the quantity. Figure 4.1 provides an overview.

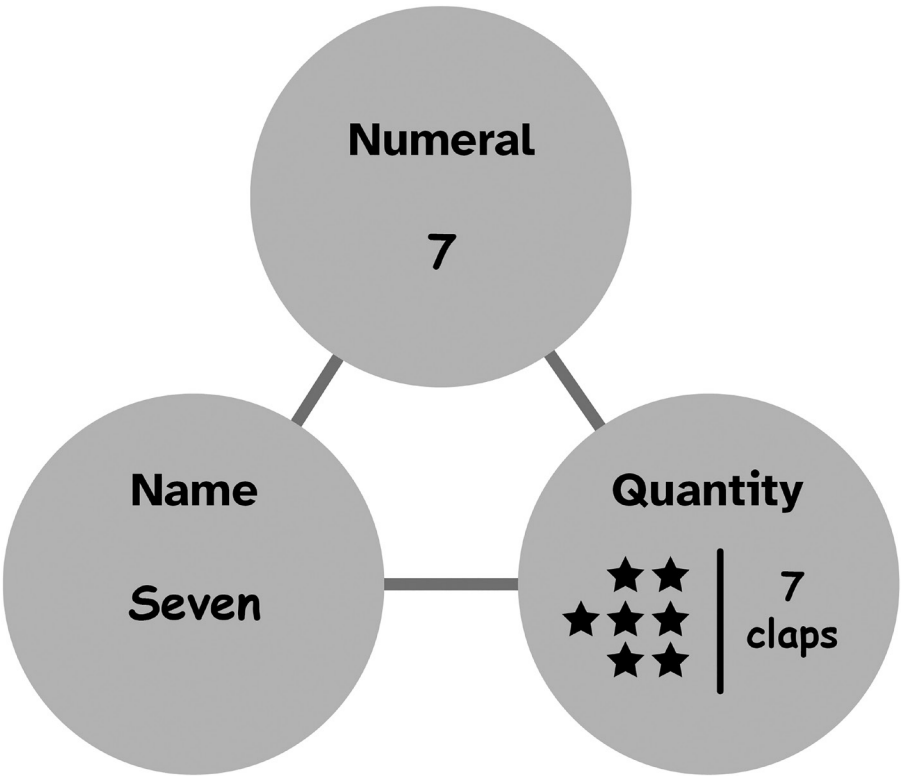


FIGURE 4.1. Three representations of a number.

While math standards are light on guidelines about which numbers students should learn about at each grade level, Figure 4.2 provides an overview (and our recommendations) of the numbers that should be practiced at each grade level. Remember, students must use and understand all three representations of number—the numeral, the name, and the quantity.

That brings us to the first math-tivity of this chapter. Think about your grade level and the students you support. In Figure 4.3 (p. 59), provide examples of the numerals, names, and quantities that are important for your students. Later in this chapter, we focus on how to help students acquire knowledge about and develop fluency with numerals.

MATH SYMBOLS

In addition to numerals, students have to understand the symbols of math. Figure 4.4 (p. 60) provides an overview of symbols named in math standards.

The list in Figure 4.4 is not exhaustive. For example, we could not determine when the greater-than or equal-to sign (≥) or less-than or equal-to sign (≤) should be introduced. For the math-tivity in Figure 4.5 (p. 61), brainstorm all the math symbols that would be important for your students to know.

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Grade	Specific Standards (NGA & CCSSO, 2010)	Number Ranges
K	• "Write numbers from 0 to 10" (p. 11) • "Represent a number of objects with a written numeral 0–20" (p. 11) • "Count to 100 by ones and by tens" (p. 11) • "Compose and decompose numbers from 11 to 19 into ten ones and some further ones" (p. 12)	0 – 100
Grade 1	• "Count to 120, starting at any number less than 120" (p. 15) • "Read and write numerals and represent a number of objects with a written numeral" (p. 15) • "Understand that the two digits of a two-digit number represent amounts of tens and ones" (p. 15)	0 – 120 (but likely to 200)
Grade 2	• "Count within 1000" (p. 19) • "Read and write numbers to 1000 using base-ten numerals, number names, and expanded form" (p. 19) • "Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones" (p. 19)	0 – 1,000
Grade 3	• "Understand a fraction" (p. 24) • Expectations with whole numbers less than or equal to 1,000 (p. 24)	0 – 1,000 w/ fractions with denominators of 2, 3, 4, 6, and 8
Grade 4	• "Recognize that in a multi-digit whole number, each digit in one place represents ten times what it represented in the place to its right" (p. 29) • "Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form" (p. 29) • Expectations with whole numbers less than or equal to 1,000,000 (p. 29)	0 – 1,000,000 w/ fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100
Grade 5	• "Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left" (p. 35) • "Read and write decimals to thousandths using base-ten numerals, number names, and expanded form" (p. 35)	0 – 1,000,000 w/ all fractions w/ decimals (to thousandths)
Grade 6	• "Understand that positive and negative numbers are used together to describe quantities having opposite directions or values" (p. 43)	-1,000,000 – 1,000,000 (and likely beyond)
Grade 7	• "Describe situations in which opposite quantities combine to make 0" (p. 49)	
Grade 8	• "Know that numbers that are not rational are called irrational" (p. 54) • "Understand informally that every number has a decimal expansion" (p. 54)	
High School	• "An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function" • "Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents" • "Represent data with plots on the real number line"	Rational numbers, irrational numbers, real numbers, complex (imaginary) numbers

FIGURE 4.2. Numbers in the standards.

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MATH-TIVITY

Numerals	Names	Quantities

FIGURE 4.3. Math-tivity: Three representations of a number for your students.

Later in this chapter, we describe strategies to help students acquire knowledge about and develop fluency with symbols. Before doing that, we spend a bit of time on the equal sign. Why? Because it's essential that students interpret the equal sign correctly. (*Note:* The equal sign was also the focus of the second author's dissertation.)

The Equal Sign (and the Inequality Symbols)

One symbol that is absolutely essential for success in math is a correct interpretation of the equal sign. Many students, across grade levels, interpret the equal sign as an operational sign that means to "do something" (McNeil, 2007; Powell, Driver, et al., 2015). Look at the equations in Figure 4.6 (p. 61). Standard equations are those with the equal sign in its standard position—second to last in an equation with three possible numbers. Many students could solve 7 + 4 = __ without a deep understanding of the equal sign. But if this standard equation was presented as 7 + __ = 11, many students might answer this as 18. They see a plus sign and an equal sign, and they "do something"—they add.

Nonstandard equations are those with the equal sign in a nonstandard position or those with more than three possible numbers. When students have an operational understanding of the equal sign, they answer 4 + 6 = __ + 5 as either 10 or 15. Unfortunately, students view the equal sign as operational well into middle and high school (Alibali et al., 2007; Knuth et al., 2006).

The equal sign should be interpreted as a relational symbol of equality, with the two sides of an equation relating to one another by sharing the same value (Jacobs et al., 2007). In grade 1, math standards focus on the equal sign by saying that students should "understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 − 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2" (NGA & CCSSO, 2010, p. 15).

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Grade	Specific Standards (NGA & CCSSO, 2010)	New Symbols
K	• "Represent addition and subtraction with equations" (p. 11)	+ (plus sign) – (minus sign) = (equal sign)
Grade 1	• "Record the results of comparisons with the symbols >, =, and <" (p. 16) • Tell and write time	< (less-than sign) > (greater-than sign) : (colon)
Grade 2	• "Using $ and ¢ symbols appropriately" (p. 20)	$ (dollar sign) ¢ (cent sign)
Grade 3	• "Apply properties of operations as strategies to multiply and divide" (p. 23) • "Understand a fraction" (p. 24) • "Measure areas by counting unit squares"	× (multiplication sign) ÷ (obelus; division sign) $\frac{a}{b}$ (vinculum; fraction bar) a / b (fraction slash) ^{2} (square sign)
Grade 4	• "Including problems in which remainders must be interpreted" (p. 29) • "Use decimal notation" (p. 31) • "An angle measure of n degrees" (p. 31)	R (remainder) . (decimal point) ° (degree sign)
Grade 5	• "Measure volumes by counting unit cubes" (p. 37) • "Use parentheses, brackets, or braces in numerical expressions" (p. 35)	^{3} (cube sign) ( ) [ ] { }
Grade 6	• "Understand the concept of a ratio" (p. 42) • "Find a percent of a quantity" (p. 42) • "Understand positive and negative numbers" (p. 43) • "Understand the absolute value of a rational number" (p. 43)	: (ratio sign) % (percent sign) – (negative sign) $\lvert a \rvert$ (absolute value sign)
Grade 7	• "Explain what a point (x, y) on the graph of a proportional relationship means" (p. 48) • "Leading to products such as (–1)(–1) = 1" (p. 48)	(a, b) (ordered pair)
Grade 8	"Know that √2 is irrational" (p. 54)	√ (square root sign)
High School	• Function • Imaginary • Factorial • Parallel • Perpendicular • Element • Subset • Union • Intersection • And • Or • Limits • Derivatives • Integrals • Summation	$f(x)$ ai n! // ⊥ ≅ ⊂ ∪ ∩ ∧ ∨ lim $x \to a$ $\frac{dy}{dx} = f'(x)$ $\int f(x)dx$ $\sum$

FIGURE 4.4. Symbols in the standards.

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MATH-TIVITY

FIGURE 4.5. Math-tivity: Math symbols for your students.

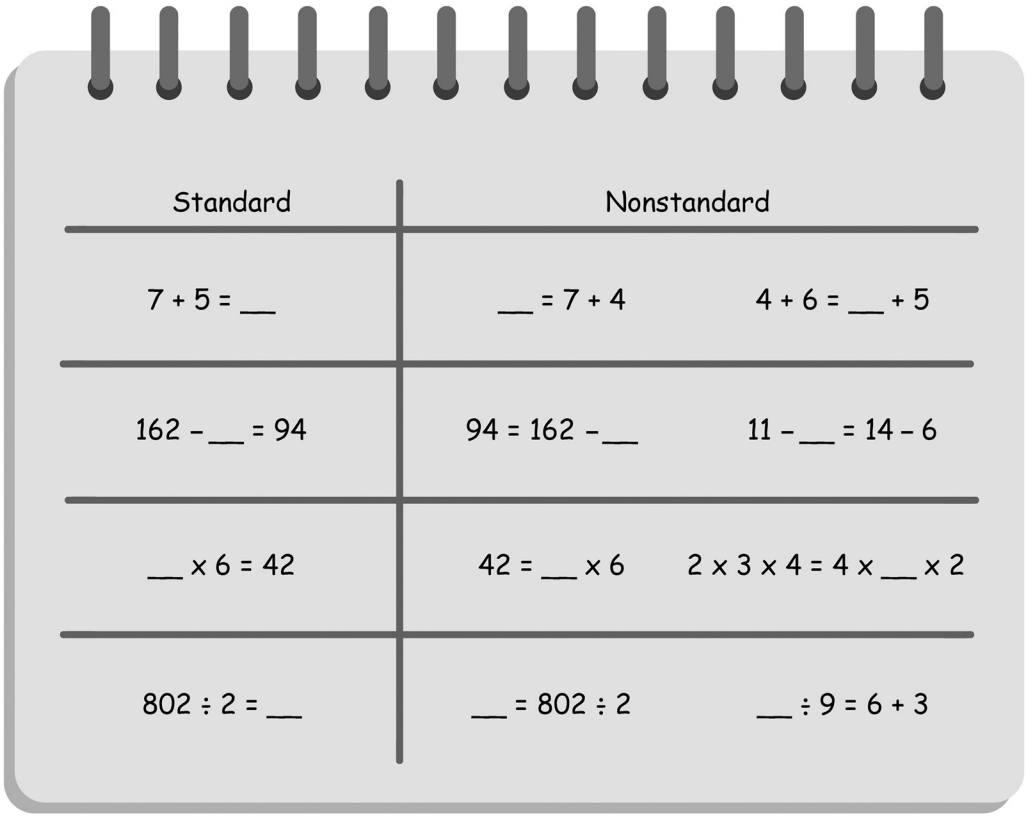


FIGURE 4.6. Standard and nonstandard equations.

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Student understanding of the equal sign (as relational) is related to later math performance (Byrd et al., 2015; Matthews & Fuchs, 2020) and can be helpful for improving word-problem performance (Powell, Berry, et al., 2021). Students pick up an operational interpretation of the equal sign in the materials used in the classroom (McNeil et al., 2006; Powell, 2012) as well as through exposure to standard equations. If students never have to interpret the equal sign as relational, particularly with nonstandard equations, they will not develop the reasoning necessary to work with the equal sign in a relational manner.

Activities with balance scales can help students interpret the equal sign as a balance between two sides of an equation. Also, helping students use language about the equal sign meaning "the same value as" can be important (Powell, Driver, et al., 2015). For example, "4 + 6 is the same value as 5 + 5." At the same time, instruction can focus on the direct relationship between the equal sign and the inequality signs of greater than (>) and less than (<). In Figure 4.7, notice how the inequality signs are two parallel lines (like the equal sign) but pinched together on either side to show greater than (>) or less than (<). If students see how similar the signs are, they should not need cute tricks like the greater gator or less-idile.

MATH VOCABULARY

As students learn the numerals of math and the symbols of math, it is vital they learn about *math vocabulary*, the terms that students use to describe math concepts and procedures. Vocabulary is used to describe the numerals (e.g., *zero*, *six*, *one hundred*, *three-fourths*, *negative eight*) and symbols (e.g., *plus*, *equal sign*, *cents*, *absolute value*) that we focused on earlier in this chapter.

At any grade level, there are over 100 or several hundred math vocabulary terms that students should know to fully participate in math. Figure 4.8 provides an overview of math vocabulary counts from a review of several math textbooks. While the

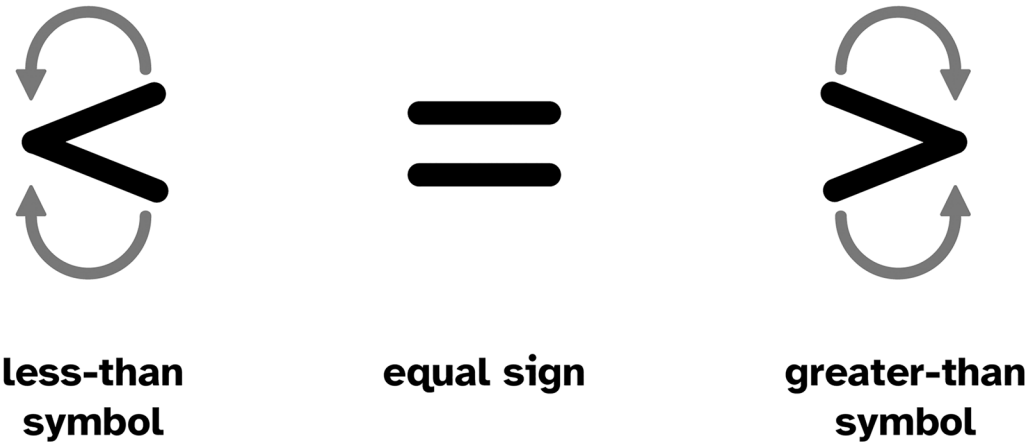


FIGURE 4.7. The equal sign (middle) and the inequality signs.

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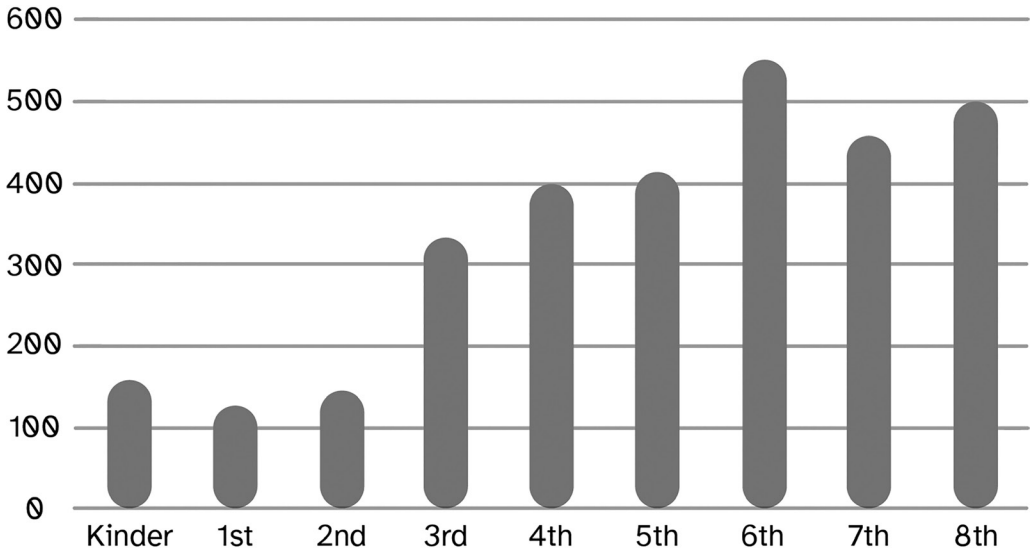


FIGURE 4.8. Vocabulary expectations across kindergarten through grade 8. From Powell, Bos, and Lin (2021).

expectations for your grade level from your math materials might vary a bit, this provides a snapshot of the number of math vocabulary terms across the elementary and middle school grades. Across these grade levels, we identified over 1,200 unique vocabulary terms that students would encounter in math by grade 8!

Math vocabulary might be difficult for students for several reasons (Rubenstein & Thompson, 2001). First, there are math terms that have different meanings in common English usage (e.g., *factor* with multiplication vs. the time *factor*). Second, there are math terms that have similar meanings in common English usage (e.g., *difference* in subtraction vs. the *difference* between North Carolina and Texas). Third, some math terms have more than one meaning (e.g., a *square* shape vs. to *square* a number). Fourth, some math terms are homophones (e.g., *sum* vs. *some*).

Beyond the terms, some students may require additional support with math vocabulary. Consider students with reading difficulties or dyslexia and students who speak more than one language (Powell, Berry, & Tran, 2020). Explicit instruction in math vocabulary is essential for many reasons.

Horizontal and Vertical Planning for Math Vocabulary

Before thinking about instruction on math vocabulary, it is necessary to understand which math vocabulary terms are important for students to learn. Keep in mind that students will interact with math terms in a number of ways, as shown in Figure 4.9. All reading in math requires an understanding of math vocabulary, as does all writing in math. As students speak and share ideas, they should use math vocabulary. Similarly,

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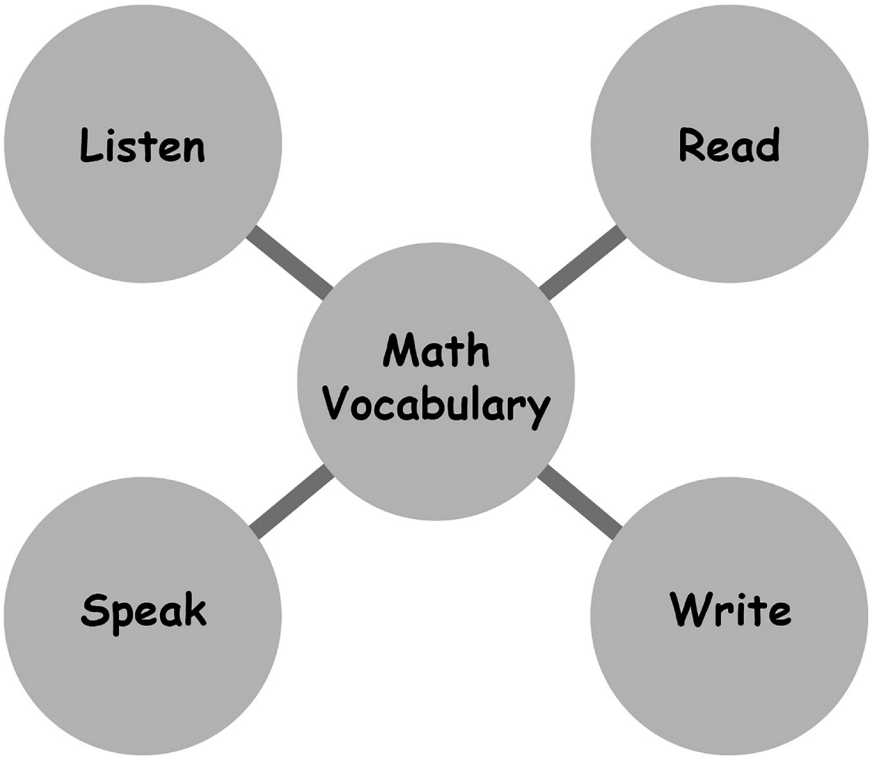


FIGURE 4.9. Ways students interact with math.

observing and listening in math—to peers, to the educator, to a video—also involves an understanding of math vocabulary.

It is important to understand the math vocabulary expectations within your grade level (i.e., horizontal planning). For the math-tivity Figure 4.10, describe ways to understand which terms are important for your students to know. Consider which resources you might consult, such as standards, district vocabulary lists, state vocabulary lists, or math textbooks.

As you think about the math vocabulary terms important at one grade level, it is also helpful to think about which terms are important across grade levels (i.e., vertical planning). You might consider:

- Which terms do students use for multiple grade levels?
- Which terms are important for success in math in high school? College? Career?
- Which terms are rooted in conceptual and procedural knowledge (and not tricks)?
- Which terms are important for strategies that students will use across grade levels (e.g., partial sums, Pythagorean theorem)?

We suggest meeting in vertical math teams to discuss essential math vocabulary across grade levels. Also, include both general education, special education, and other support personnel (e.g., math interventionist, math specialist, school psychologist) in

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FIGURE 4.10. Math-tivity: Math vocabulary terms important for your students.

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these conversations to ensure that math vocabulary terms are consistently used across settings within the school day.

Consistent Use of Formal Math Vocabulary

Once educators have decided on which math vocabulary terms are important for their students to know, it is essential to make sure these vocabulary terms are focused on the formal language of math (Hughes et al., 2016; Powell et al., 2019). The examples in Figure 4.11 compare formal math terms to colloquial, informal terms.

What are terms that your students use informally when formal math terms should be used? That's what we would like you to explore in the next math-tivity. In Figure 4.12, provide at least five examples of using formal terms instead of informal terms. For example, "say *vertex* instead of *corner*" or "say *vertical* instead of *the line that goes up and down*."

Definitions and Precision with Formal Math Vocabulary

It is imperative for students to have accurate and concise definitions of the formal terms. While it is fabulous if a student knows the term *denominator*, that knowledge has to be accompanied by knowing that a denominator represents a length, area, or set divided into equal parts. Both the formal term *and* its meaning are integral to proficiency in math vocabulary.

Figure 4.13 shows examples of math vocabulary terms that students might often confuse. The question is, if a student knows the terms *constant*, *variable*, *term*, and *coefficient*, do they also know a definition of each of those terms, and can they use those terms with precision?

In Figure 4.14, think about math vocabulary terms that students might encounter within the same unit or chapter. Think about whether students have precise definitions of those terms and can explain the similarities and differences among them. Generate up to four groups of terms. Consider the examples provided in Figure 4.13.

Example	Say this …	Instead of this …	Why?
$\begin{array}{r} 34 \\ -\ 18 \\ \hline 16 \end{array}$	*minuend*	bigger number	The minuend won't always be greater or a number.
	subtrahend	smaller number	The subtrahend won't always be lesser or a number.
	difference	answer	Signify that this resulted from subtraction

Example	Say this …	Instead of this …	Why?
15 + 15 = 30	*addend*	first number	The commutative property shows order doesn't matter.
	addend	second number	The commutative property shows order doesn't matter.
	total	answer	Signify that this resulted from addition

Example	Say this …	Instead of this …	Why?
$\frac{3}{4} = \frac{6}{8}$	*numerator*	top number	Communicates the fraction's overall value, based on relationship to the denominator
	denominator	bottom number	The denominator provides the value per whole.
	equivalent	same as	These are not the exact same fraction, so the values are equivalent.

Example	Say this …	Instead of this …	Why?
$3x^2$	*coefficient*	number	As "co," as the 3 belongs with the variable to the power of 2, meaning there are three x^2.
	variable	times	The value for x is undetermined from this term.
	exponent	little or upper number	This number identifies how many times the base is multiplied by itself.

FIGURE 4.11. Comparing formal math terms to informal terms.

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MATH-TIVITY

Say this...	Instead of this...

FIGURE 4.12. Math-tivity: Examples of formal math terms versus informal terms.

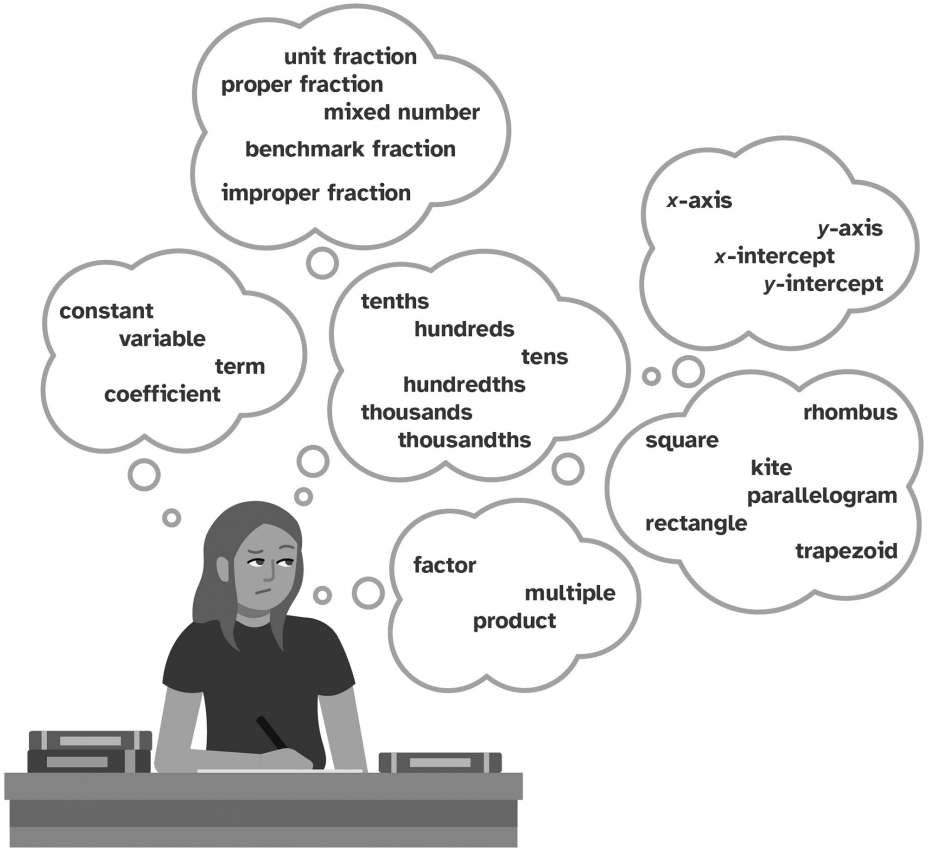


FIGURE 4.13. Examples of math vocabulary terms that students might confuse.

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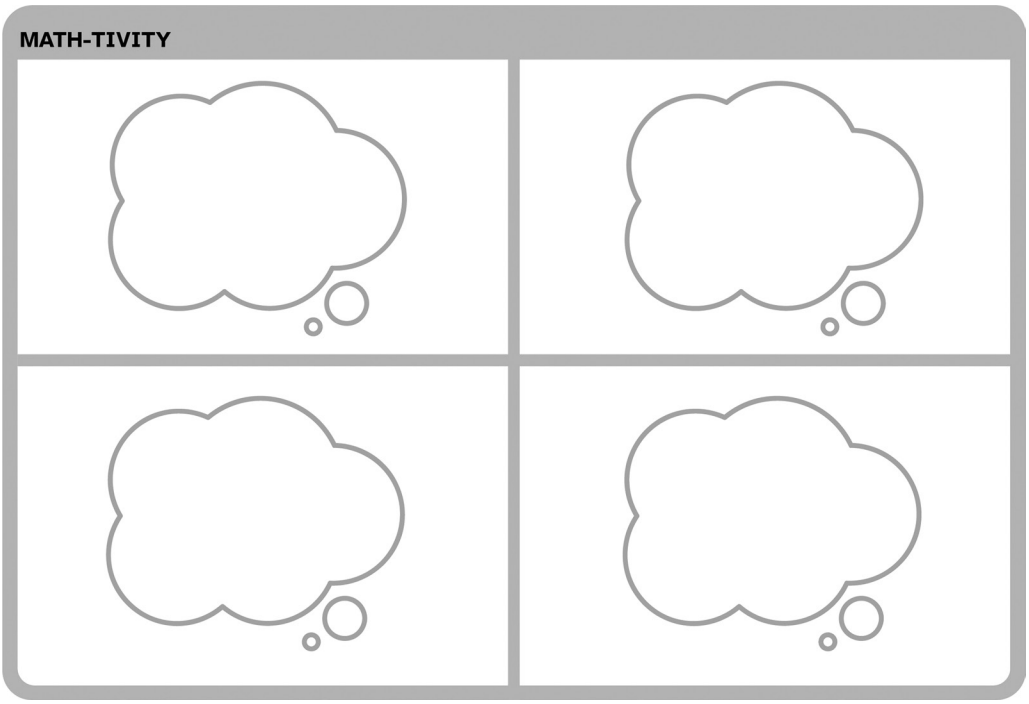


FIGURE 4.14. Math-tivity: Groups of terms that require precision.

ACQUISITION OF NUMBERS, SYMBOLS, AND VOCABULARY

To help students acquire knowledge related to numerals, symbols, and vocabulary, explicit instruction is necessary (Stevens & Mowbray, 2024). One resource that might be helpful for such instruction is the use of a vocabulary map like the one in Figure 4.15. Notice how this vocabulary map connects vocabulary terms to important symbols or numerals. It also connects a target vocabulary term to related terms, provides a visual representation for the term, and provides an opportunity for students to apply their knowledge of the term.

In addition to explicit instruction, it is helpful to have classroom resources that emphasize numerals, symbols, and vocabulary. Word wall cards (see Figure 4.16) could be used, and such cards should always include the term, a student-friendly definition, and a visual that explains the term (Stevens et al., 2023). Cards or similar resources are only helpful when they are regularly referenced. For example, educators should encourage students to use terms from the word wall as they do a turn-and-talk with a partner about how they solved a problem or provide a written explanation for solving a problem.

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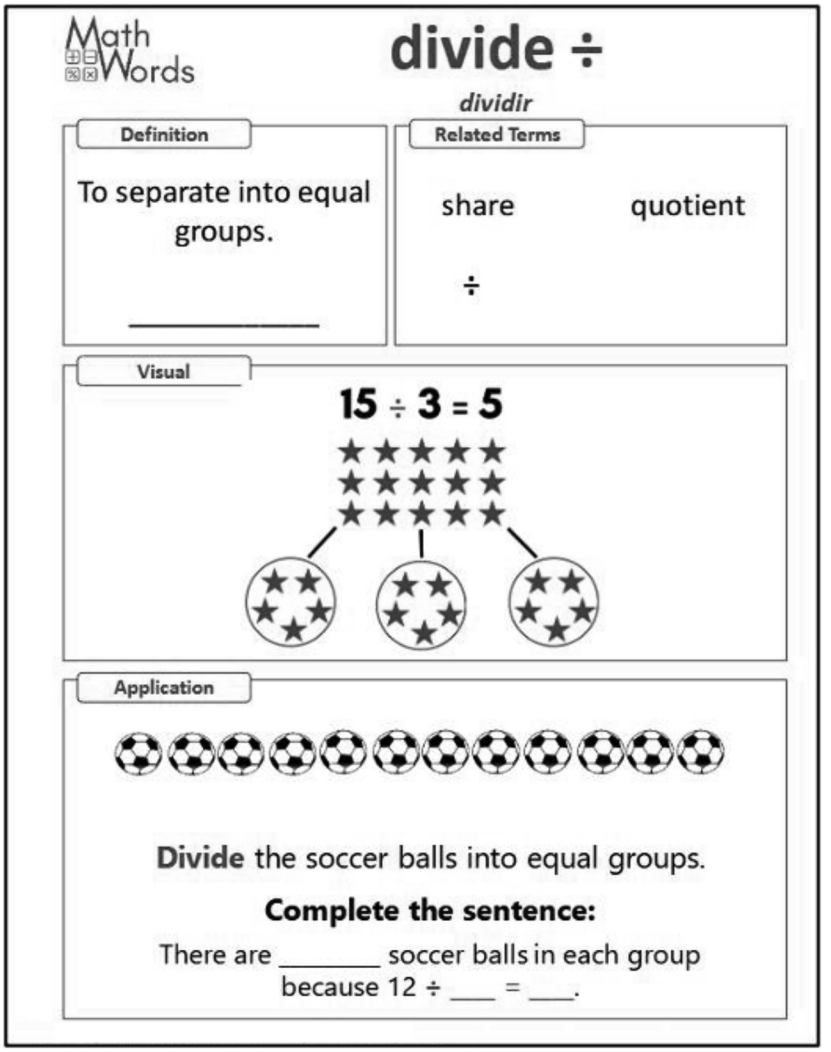


FIGURE 4.15. Vocabulary map. Copyright © 2024 ProjectMathWords.com. Do not reprint without permission.

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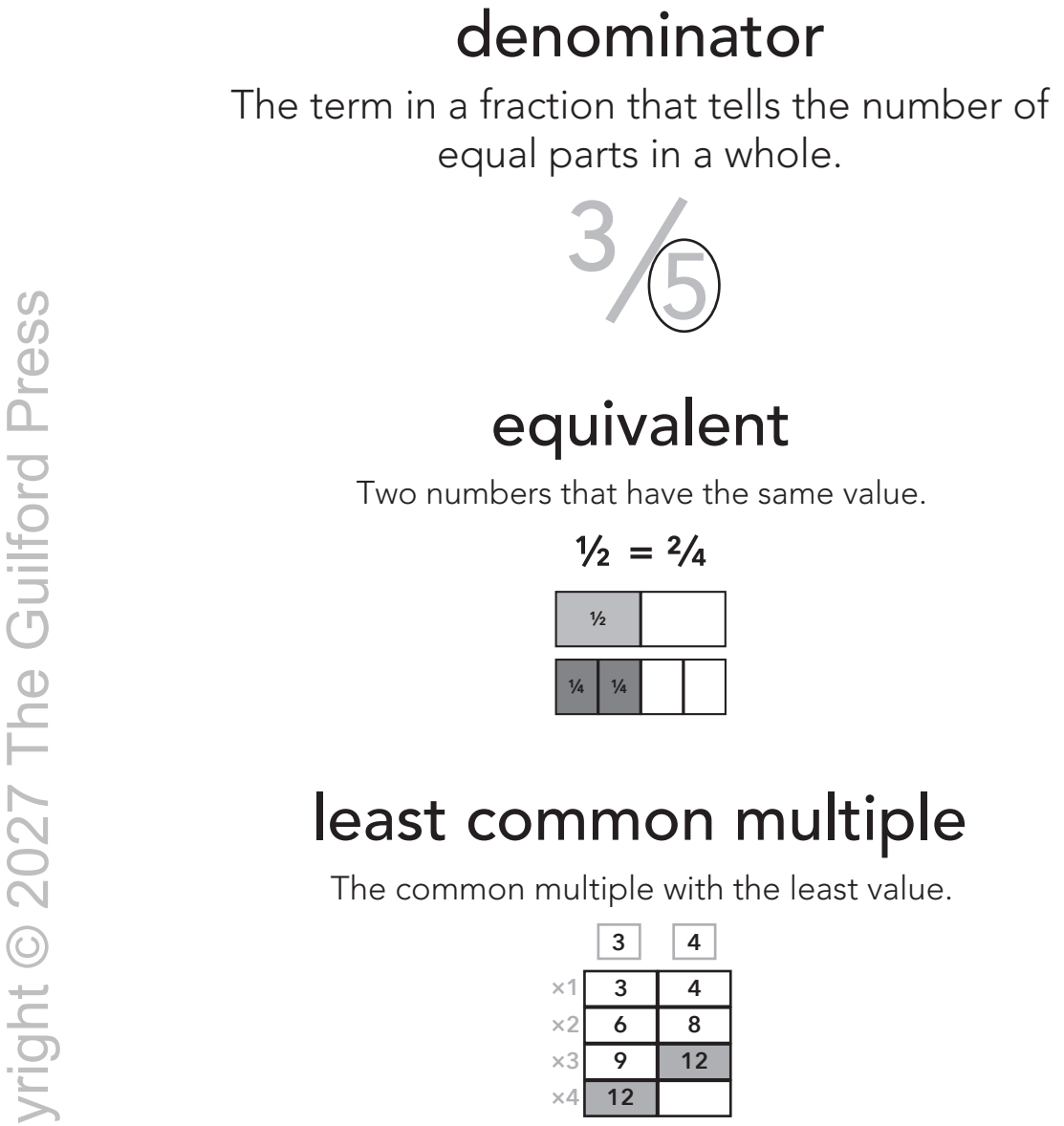
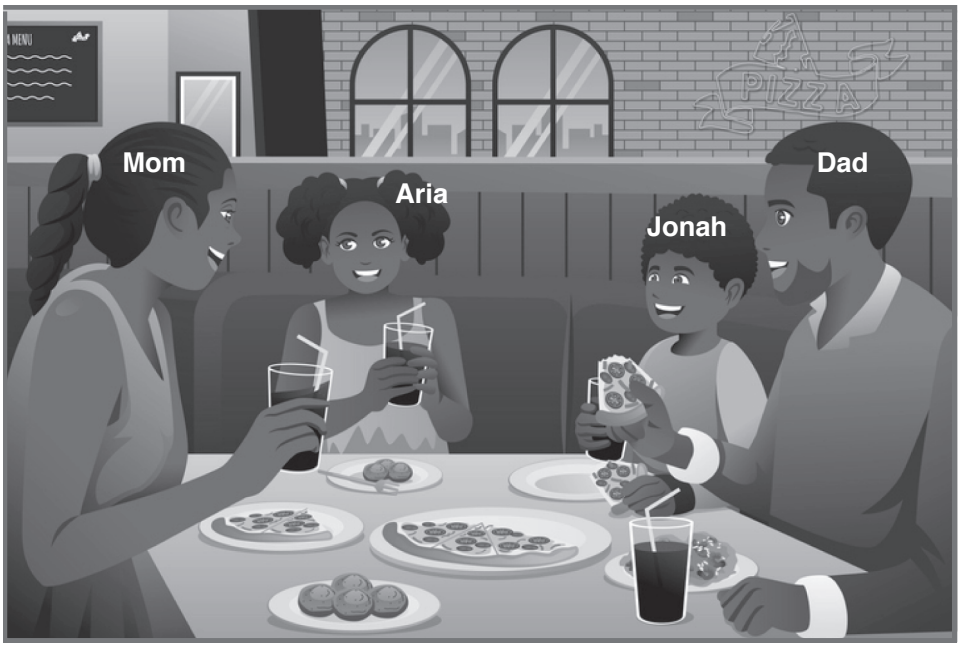


FIGURE 4.16. Example word wall cards.

Storybooks are useful for explicitly introducing and teaching specific vocabulary terms (Hassinger-Das et al., 2015). Hands-on representations, virtual representations, or other visuals (e.g., drawings) can support the acquisition of numbers, symbols, and vocabulary terms (Lariviere et al., 2025). Additionally, to help students with the acquisition of numbers, symbols, and vocabulary terms, students could create their own glossaries of vocabulary terms (Nelson & Kiss, 2021).

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STORY SLAM - PIZZA



Read the story and choose the correct math term for each blank.

Jonah and Aria are eating pizza with their mom and dad.

Aria loves garlic rolls and takes ___ **(1, 3)** out of the 7 total rolls for her plate.

Aria has a _____ **(area, fraction)** of the garlic rolls.

Aria's fraction of garlic rolls is ____ **(3/7, 5/8)**.

The family orders a large pizza that is ____ **(perpendicular, partitioned)** into 7 equal slices.

Aria notices that 7 is the_____ **(numerator, denominator)** because there are 7 slices in the whole pizza.

If Mom eats 2 slices of pizza, 2 is the _____ **(numerator, denominator)** because 2 is part of the whole pizza.

Mom's fraction of the pizza is ____ **(6/10, 2/7)**.

Aria _____ **(compares, regroups)** 3/7 and 2/7.

3/7 and 2/7 are not _____ **(fractions, equivalent)**.

3/7 is _____ **(greater than, less than)** 2/7.

Fractions are delicious!

Math Words

FIGURE 4.17. Example story slam. Copyright © 2024 ProjectMathWords.com. This is a copyrighted work and should not be reproduced, shared, or otherwise disseminated (including digitally) without permission.

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FLUENCY WITH NUMBERS, SYMBOLS, AND VOCABULARY

As students acquire knowledge of numbers, symbols, and vocabulary, it is important for them to participate in many different activities to help build and maintain fluency—that ease and accuracy with numbers, symbols, and vocabulary (Riccomini et al., 2015). Activities could include the use of flash cards (Petersen-Brown et al., 2019) or playing a matching game in which students match a term with a definition or visual (Lin & Powell, 2023). Figure 4.17 provides an example of a fluency-building activity called a *story slam*. Regular (i.e., daily) practice is necessary for students to develop fluency.

For the math-tivity in Figure 4.18, provide several strategies that you would use to build fluency with numbers, symbols, and vocabulary terms.

MATH-TIVITY

FIGURE 4.18. Math-tivity: Fluency building with your students.

YOUR STRENGTHS AND OPPORTUNITIES FOR GROWTH

As we conclude this chapter, look at the math-tivity in Figure 4.19. Think about the information within this chapter. Describe your strengths with this content. Describe your opportunities for growth with the content.

MATH-TIVITY

Strengths:

Opportunities for Growth:

FIGURE 4.19. Math-tivity: Reflection on Chapter 4.

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