Chapter 13

Other Horizons

To accomplish great things,
we must dream as well as act.

—Anatole France

Overview

- Interaction and curvilinear effects
- Multilevel structural equation models

Interaction and curvilinear effects

- There are two general ways to estimate interaction effects in SEM:

  1. *Analyze a model across multiple samples:* If unstandardized estimates differ appreciably across the samples, then we conclude that group membership moderates those effects

  2. *Analyze a model in a single sample with product terms specified by the researcher:* This same strategy can be used to estimate curvilinear relations (trends)

- These two approaches can be combined—for example, it is theoretically possible to analyze a model with product terms across multiple samples

Interaction and curvilinear effects

- Interaction or curvilinear effects of observed variables are represented by product terms that are entered along with the original variables in a statistical model
- This is the method used in multiple regression
- It also underlies the estimation of interaction and trend effects in the analysis of variance (ANOVA) and in path analysis (e.g., Baron & Kenny, 1986; Lance, 1988)

Interaction and curvilinear effects

- For example, the power term X^2 represents the quadratic relation of X to Y when both X and X^2 are entered as predictors of Y in the same regression equation

- Likewise, the term X^3 represents the cubic relation of X to Y when X, X^2, and X^3 and are all entered as predictors of Y in the same equation

- However, note that it rarely necessary to estimate nonlinear effects beyond quadratic ones in behavioral data

Interaction and curvilinear effects

- Example: The data in Table 13.1(a) were analyzed in the Curve Estimation module of SPSS, and the results both without and with X^2 in the equation (respectively, LIN and QUA) are summarized here:

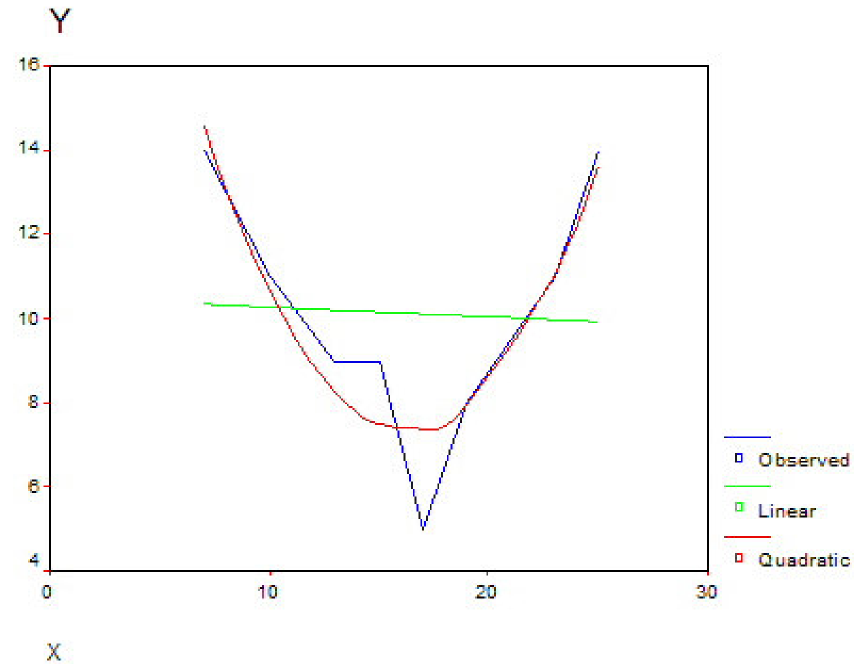
```
Curve Fit

MODEL:  MOD_1.

Independent:  X

  Dependent Mth   Rsq  d.f.       F  Sigf      b0      b1      b2

   Y        LIN  .002     6     .01  .913 10.4951  -.0230
   Y        QUA  .860     5   15.33  .007 29.4861 -2.7142   .0832
```



Interaction and curvilinear effects

- Suppose that *X* and *W* are two continuous predictors of *Y*
- The product term *XW* represents the interactive relation between *X* and *W* when *X*, *W*, and *XW* are all entered in the same regression equation as predictors of *Y*
- This interactive relation is a linear × linear one where the linear association between *X* and *Y* changes uniformly (in a linear way) across the levels of *W*

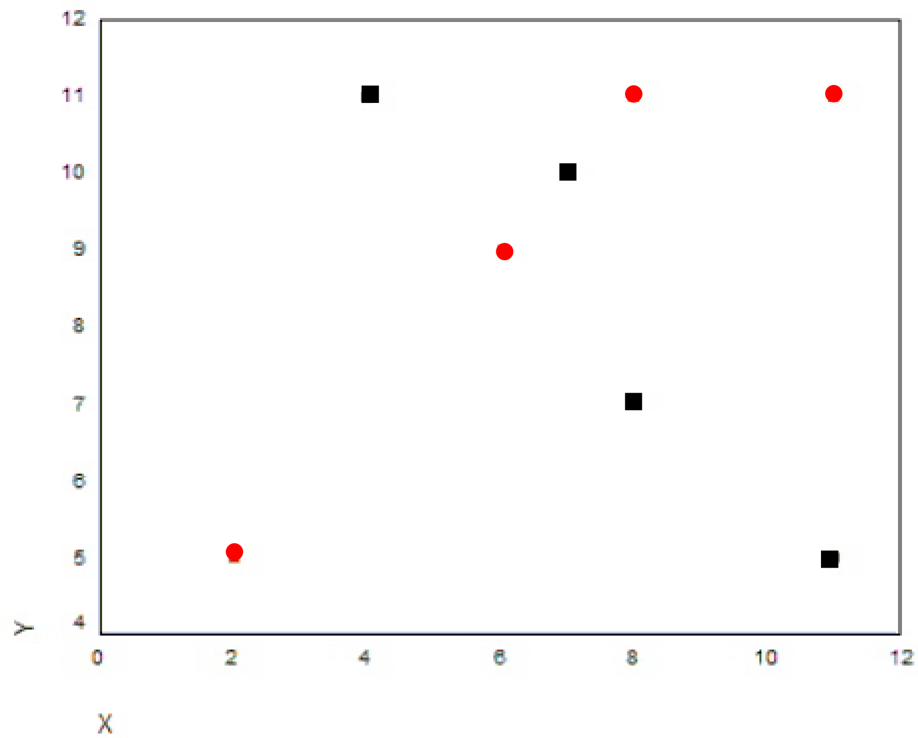
Interaction and curvilinear effects

- Because interactive effects are joint, it is also true that *XW* represents a uniform change in the association between *W* and *Y* across the levels of *X*

- The estimation with multiple regression of interaction effects by entering product terms in the equation is known as *moderated multiple regression*

Interaction and curvilinear effects

- Example: The scatterplot for *X* and *Y* based on the data in Table 13.1(b) is presented here
- Cases with low scores (≤ 16) on *W* are shown as red dots, and cases with high scores on *W* are shown as black squares
- For low scores on *W* (red dots), the relation between *X* and *Y* is positive; just the opposite is true for high scores on *W* (black squares):



Interaction and curvilinear effects

- Even higher-order interactions can also be represented with product terms
- For example, the product term XW^2 represents a linear × quadratic interaction
- This means that the linear relation of *X* to *Y* changes faster at higher (or lower) levels of *W*

Interaction and curvilinear effects

- Because the estimation of higher-order interactive or curvilinear effects may require the analysis of numerous product variables, very large samples may be necessary
- See J. Cohen, P. Cohen, West, and Aiken (2003) for more information about the estimation of curvilinear and interaction relations with multiple regression

Interaction and curvilinear effects

- In the *indicant product approach* in SEM, product terms are specified as indicators of latent variables that represent interactive or curvilinear effects of latent variables

- Suppose that

    1. factor *A* has two indicators, X_1 and X_2

    2. the reference variable for *A* is X_1 (i.e., $A \rightarrow X_1 = 1.0$)

    3. there is one criterion variable, *Y*

Interaction and curvilinear effects

- The equations of the measurement model for the indictors of *A* are:

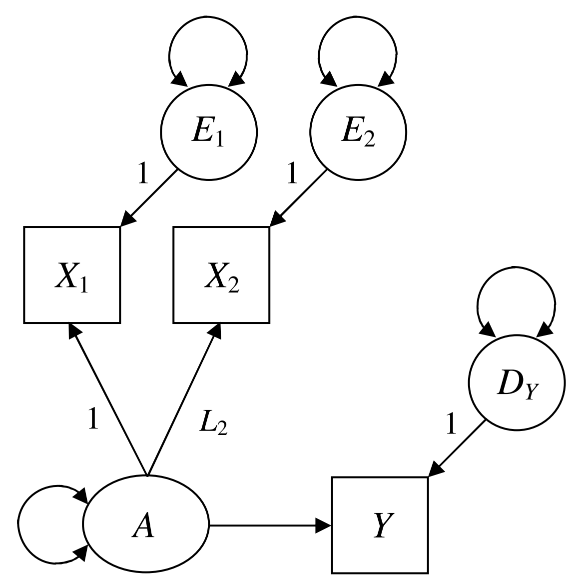
$$X_1 = A + E_1$$

$$X_2 = L_2 A + E_2 \qquad (1)$$

where L_2 is a freely-estimated factor loading

- The diagram for the whole model is presented here (Figure 13.1(a)):

(a) Linear effect only



Interaction and curvilinear effects

- In the model just presented, the coefficient for the path $A \rightarrow Y$ estimates the linear effect of A on Y
- To estimate the quadratic effect of A, it is necessary to add to the model the latent product variable A^2
- The indicators of A^2 are the product terms X_1^2, X_2^2, and $X_1 X_2$
- The product term $X_1 X_2$ is not here an indicator of an interaction effect because its components, X_1 and X_2, are specified to measure the same construct

Interaction and curvilinear effects

- By squaring or taking the product of the corresponding expressions in Equation 1, the equations of the measurement model for the product indicators of A^2 are:

$$X_1^2 = A^2 + 2AE_1 + E_1^2$$

$$X_2^2 = L_2^2 A^2 + 2L_2AE_2 + E_2^2 \qquad (2)$$

$$X_1 X_2 = L_2A^2 + L_2AE_1 + AE_2 + E_1E_2$$

Interaction and curvilinear effects

- Equation 2 shows that the measurement model for the product indicators involves not just the latent product factor A^2 but also five additional latent product terms:

$$AE_1,\ AE_2,\ E_1^2,\ E_2^2,\ \text{and}\ E_1E_2$$

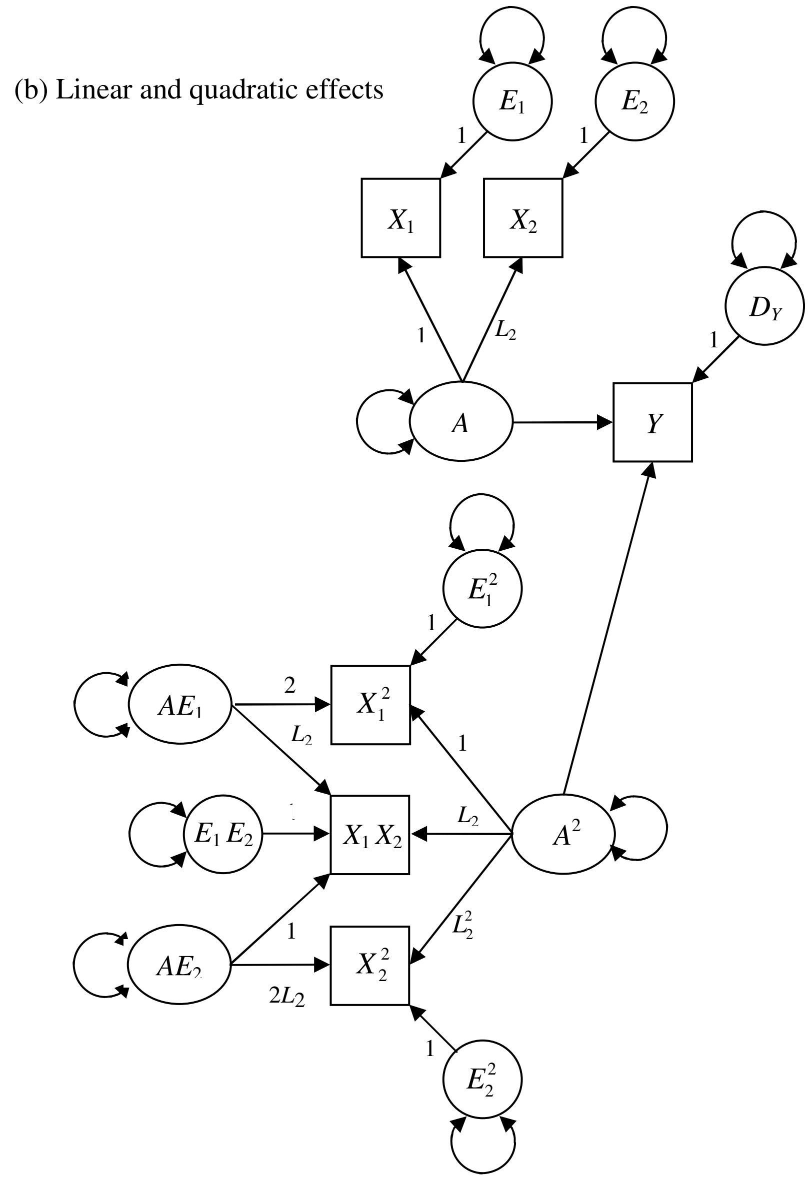
- Note in Equation 2 that all of the factor loadings are either constants or functions of L_2, the loading of X_2 on A (Equation 1)

- Other parameters that correspond to Equation 2 include the variances of A^2 and those of the five other latent product terms

Interaction and curvilinear effects

- Presented here is the diagram for the model that includes factors *A* and A^2 (Figure 13.1(b)):

- The coefficient for the path $A \rightarrow Y$ estimates the linear effect, and the coefficient for the path $A^2 \rightarrow Y$ estimates the quadratic effect



Interaction and curvilinear effects

- Kenny and Judd (1984) were among the first to describe a method for estimating structural equation models with product indicators
- This method assumes normal distributions and means that equal zero for the nonproduct variables
- Under these assumptions, the parameters of the measurement model for the product indicators (e.g., Equation 2) are functions of the parameters of the measurement model for the original indicators (e.g., Equation 1)

Interaction and curvilinear effects

- For example, Kenny and Judd (1984) showed that the variance of the latent quadratic factor A^2 equals two times the squared variance of the latent linear factor A
- With these and other nonlinear constraints imposed on parameters of the measurement model for the product indicators, it is then possible to estimate the effects of interest, such as $A^2 \rightarrow Y$
- A drawback to the Kenny-Judd method is that not all SEM computer programs support nonlinear constraints
- Also, correctly programming all such constraints can be tedious and error-prone (e.g., Jöreskog & Yang, 1996)

Interaction and curvilinear effects

- Ping (1996) described a two-step estimation method that does not require nonlinear constraints, which means that it can be used with just about any SEM computer program

- It requires essentially the same basic statistical assumptions as the Kenny-Judd method

- In the first step of Ping's method, the model is analyzed *without* the product indicators (e.g., Figure 13.1(a))

- One records parameter estimates from this analysis and calculates the values of parameters of the measurement model for the product indicators implied by the Kenny-Judd model

Interaction and curvilinear effects

- These values can be calculated either by hand or using a set of templates for Microsoft Excel created by Ping that can be freely downloaded from:

  http://home.att.net/~rpingjr/

- These calculated values are then specified as fixed parameters in the second step where all variables, product and nonproduct, are analyzed

- Path coefficients for curvilinear or interactive effects of latent variables are obtained in the second analysis

Interaction and curvilinear effects

- Some technical problems can up with either the Kenny-Judd or Ping methods (Rigdon, Schumaker, & Wothke, 1998)
- One of these is multicollinearity—this is because correlations between product terms and their constituent variables can be high
- One way to deal with this problem is to c*enter* the original indicators before calculating product terms, which means to set the averages of the original indicators to zero

Interaction and curvilinear effects

- Iterative estimation may be more likely to fail for models with many product variables
- Ping (1995) suggested some "shortcuts" that involve analyzing fewer product variables if unidimensional measurement can be assumed for the original indicators

Interaction and curvilinear effects

- The assumption of normal distributions for the original indicators is crucial for both the Kenny-Judd and Ping methods
- If this assumption is not tenable, then parameter estimates implied by the Kenny-Judd method (which also determine estimates in Ping's method) may not be accurate
- However, distributions of product indicators may be severely nonnormal even if those of the original indicators are generally normal

Interaction and curvilinear effects

- This means that normal-theory estimation methods, such as maximum likelihood (ML), may not yield accurate results
- There are methods for analyzing nonnormal data, but they can be difficult to apply to models with product indicators (Jöreskog & Yang, 1996)

Multilevel structural equation models

- *Multilevel models* are also known as *random coefficient models* or *covariance components models*

- These terms refer to classes of statistical models for hierarchical data where individuals are clustered into larger units, such as siblings within families or workers within departments

- Within each level, scores may not be independent

- Repeated measures data sets are also hierarchical in the sense that multiple scores are clustered under each case

Multilevel structural equation models

- Traditional statistical methods generally deal with a single unit of analysis only
- They also typically assume independence of the observations
- Applied to clustered data, traditional methods may require either aggregation or disaggregation of the scores in a way that ignores potentially important information

Multilevel structural equation models

- An example of a statistical technique for hierarchical data is *two-level regression*

- In this approach, there is a level-1 equation for variables measured at the case level, such as achievement scores and demographic characteristics of individual students

- There is also a level-2 equation for data about higher-order units, such as characteristics of schools attended by the students or the neighborhoods in which they live

- The level-1 equation would be fitted to a pooled within-groups covariance matrix, and the level-2 equation would be fitted to a between-groups covariance matrix

Multilevel structural equation models

- Unlike standard (single-level) regression, which assumes independence over all *N* scores, in two-level regression, independence is assumed only over the highest unit of analysis (e.g., schools)

- Through the simultaneous analysis of both equations in a two-level regression, the effects of level-1 variables and level-2 variables can be accurately estimated

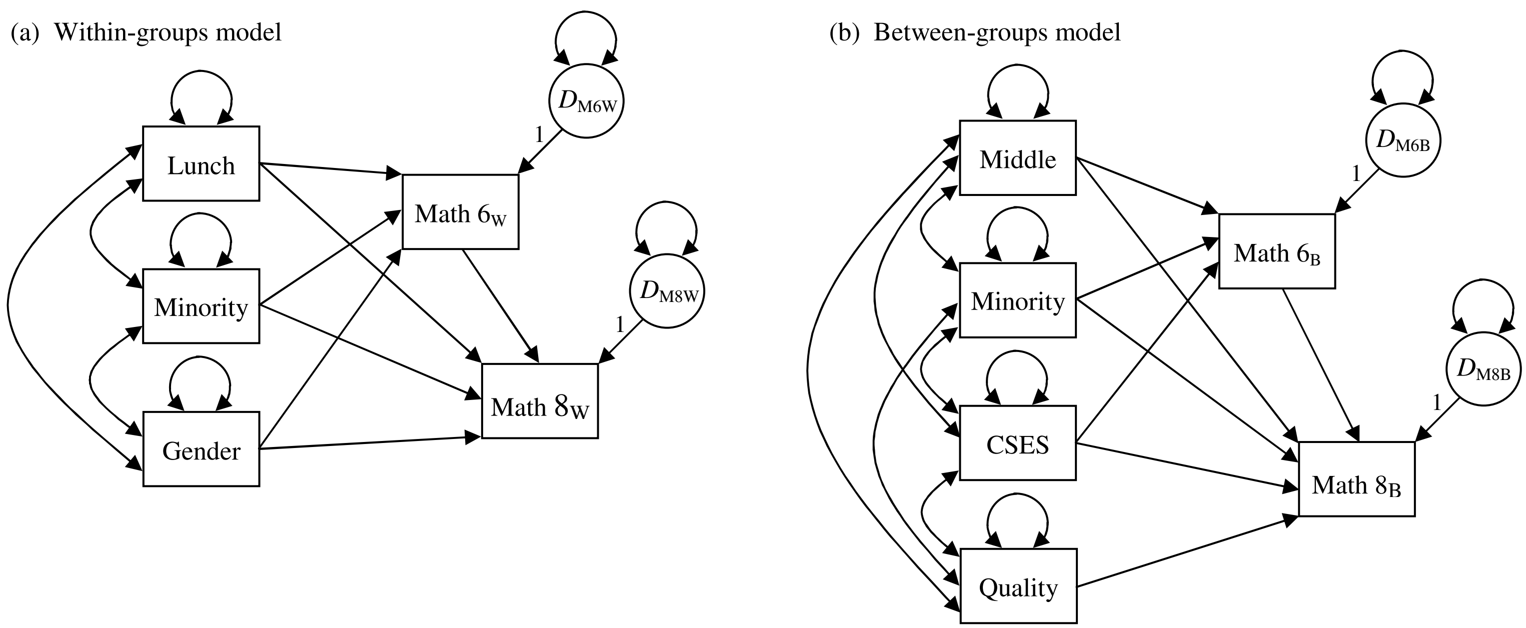
- In contrast, the use of single-level regression to analyze clustered data may not yield accurate results because of violation of the assumption of independence over all cases (e.g., Osborne, 2000)

Multilevel structural equation models

- Mediator (indirect) effects can be estimated in a *two-level path analysis*
- One model analyzed in two-level path analysis is a within-groups structural model of observed variables for case-level variables
- Another model analyzed is a between-groups model for higher-order variables (i.e., those under which cases are clustered)
- Each model just described can be analyzed either simultaneously or separately with a computer program for SEM that supports multilevel analyses

Multilevel structural equation models

- Example: In a two-level path analysis, Heck (2001) analyzed these within-groups and between-groups models of math achievement over grades 6 to 8 in a large sample of students who attended over 50 different schools (Figure 13.2)
- The within-groups model concerns characteristics of students, and the between-groups model concerns characteristics of schools, such as community socioeconomic status (CSES):



Multilevel structural equation models

- A drawback of either single-level path analysis (chaps. 5-6) or multilevel path analysis is the inability to

    1. take direct account of measurement error

    2. represent the measurement of constructs with multiple indicators

Multilevel structural equation models

- Fortunately, there are also multilevel versions of confirmatory factor analysis (CFA) models and structural regression (SR) models
- These models can also optionally include a mean structure
- A latent growth model (LGM) is a special kind of two-level model in that scores on repeated measures variables are clustered within individuals (chap. 10)

Multilevel structural equation models

- Most examples of the analysis of multilevel models in the SEM literature are two-level models where level 1 concerns case variables and level 2 concerns group variables

- Examples can be found in

  - Kaplan (2000, chap. 7), who describes the analysis of a two-level CFA model of student perceptions of school climate

  - T. Duncan et al. (1997), who analyzed a multilevel LGM of levels of substance use by adolescents over a four-year period

  - Rosser, Johnsrud, and Heck (2003), who estimated a multilevel SR model of the effectiveness of academic administrators

Multilevel structural equation models

- Until recently, it was rather difficult to analyze multilevel structural equation models
- This is mainly because most SEM computer programs were intended for single-level analyses only
- However, more recent versions of some traditional programs for SEM, such as EQS and LISREL, now feature specific syntax for multilevel analyses
- The Mplus program is very flexible in that it can analyze multilevel models with either continuous or categorical latent variables

Multilevel structural equation models

- The SEM computer programs just mentioned may use special forms of ML estimation for multilevel data that adjust for unequal group sizes
- However, it can still be challenging to correctly specify and analyze a multilevel model with numerous within-groups and between-groups predictors

Multilevel structural equation models

- Also, methods for multilevel SEM are still evolving, which means that there are fewer guidelines for interpreting and reporting the results of a multilevel SEM

- For now, it would be worth the effort for researchers who are familiar with the basics of SEM and also analyze hierarchically-structured data to learn more about multilevel SEM and related statistical techniques

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